Cross-media externalities in advertising markets*

CERNA Working Paper

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February 27, 2021

Abstract

Online advertising has been growing significantly over the past few years. Targeting capabilities allowed by the digital economy now seem to overshadow traditional mass-media advertisement. Yet, advertising is a semantic good whose effectiveness is subject to contextual externalities, including the effect of other ads. This paper questions the trends in online substitutability over offline by considering externalities between both ecosystems. We propose an empirical study which quantifies how online ad effectiveness (i.e clicks) is subject to externalities from other media (e.g. TV, radio, display). Using data from three advertisers, we demonstrate a positive effect of traditional media on online ads effectiveness. We place our results in the context of regulatory concerns already at stake regarding online platforms and advertising. In particular, we highlight an ambiguous function of online advertising which is both to *advertise* and *distribute* goods.

Keywords— Advertising, externalities, pricing, regulation

*The authors are very grateful to Albane Demurger, summer intern, for her great help on the econometric analysis. This paper also benefited from useful comments from CERNA colleagues.

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1 Introduction

On October 25th 2020, a New York Times article revealed Google pays Apple billions-of-dollars to be featured as the default search engine on iPhone's Safari (NYT, 2020). This secret agreement – retitled the "*Deal That Controls the Internet*" – ensures half of Google's traffic, resulting in as much revenues for the firm. Indeed, by securing users queries, Google asserts at the same time the visibility of search ads on which it earns revenues. Yet, many advertisers have been complaining of this pay-per-query model in which Google charged them every time a consumer looked up their name.

Online advertising is at the heart of this business. It has been growing significantly over the past decade to such an extent it now accounts for the majority of firms' media spending (eMarketer, 2019). The decrease in targeting cost is the rational for this trend. However, the story related by the New-York Times may give reasons to believe otherwise. The online advertising industry revolves around *directly observable* consumers' responses to ads, e.g. effective impressions, clicks, conversions. These metrics serve as an instrument to measure and price the effectiveness of online ads. Yet, online consumers' responses may be subject to external effects, in particular those generated by other media. Indeed, an online ad is likely to benefit from bandwagon effects or interest peaks initiated by offline mass-media, for the benefit of the ad-seller. This paper empirically quantifies the existence of external effects between advertising media at the firm-level.

Our research applies a microeconomic framework to the empirical advertising effectiveness literature. It enlightens the traditional measurement problems with assumptions on advertising's industrial organization and actors behavior regarding externalities.

While the emergence of online ads has involved new research on the substitutability and effectiveness of both traditional and targeted advertisement, the question of externalities between media remained an under-explored area. We test our assumptions with an empirical study based on data from three multi-media advertisers operating in two distinct markets. While most models focus on sales, we use a Poisson regression to study the impact of different media (e.g. TV, radio, display) on online ad outcomes (e.g. Google clicks).

Our study demonstrates offline ad investments have a positive impact on online ad effectiveness. For example, increasing the stock of radio ad investment by 1% ($\simeq$ 126€) rises clicks on Google ads by 1.5% ($\simeq$ +368 click). In opposite, the externalities online ads impose on each others are often null perhaps negative.

It seems to us digital media benefit from a positive context generated by offline investment's stock *via* their pay-per-action pricing models. This may be a consequence of the fact that online advertisement differs from a traditional one by providing an access to the advertised goods. The existence of such external effects uncompensated (perhaps *captured*) by online media raises reflection about the pricing and regulation of online ad selling.

The present paper is organized as follow. In Section 2, we set the framework of our study based on microeconomic and quantitative marketing literature. Section 3 presents an econometric model of cross-media externalities. The results discussed in Section 4 tend to demonstrate mostly positive externalities between advertising media, confirming the presence of cross-media effects. We then discuss the implications for advertisers, media and regulators in Section 5.

2 Conceptual framework

2.1 Advertising and its externalities

Advertising has been, and is still, hotly debated among economists. It has long been considered either persuasive or informative – Bagwell (2007) made a complete review of this controversy. These dichotomy, considered too restrictive on consumer's behavior, led Becker and Murphy (1993) to develop an alternative theory in which advertising is a commodity itself, complementary to the advertised product. The ad is a good when it rises consumer's marginal utility for the product, otherwise, it is a bad. This theory considers the external effects of advertising are internalized into consumers' preferences whom will (de)value the product. Advertising is thus a *media complement* of the good – its social representation – which produces a meaning consumers may find useful or not.

This idea of *ad complement* found an empirical representation in the concept of *adstock*, or *advertising goodwill*, early theorized by Nerlove and Arrow (1962). The intuition is that advertising enters consumer's utility function beyond its diffusion period. It is formalized as the stock of ad expenditures a, depreciated at a rate $(1-\delta)$: $A_t = a_t + \delta A_{t-1}$. Adstock is a strong economic assumption: it quantitatively catalyzes the inter-temporal effects of advertising. As such, it has a wide range of econometric specification (Joy, 2006). Yet, a core hypothesis of empirical adstock is that an ad's effect over time tends to be null. However, factually, Apple's 1984 ad and its media coverage have built a myth[1] whose resonance still participates in the notoriety of the brand and its latter campaigns among consumers Isaacson (2011). While adstock theoretically captures the notorious and imaginary dimension of the good, its empirical representation – by assuming monotonous returns – is an imperfect abstraction of reality.

In addition to being inter-temporal, the advertising good can also be complementary to other products. Advertising literature refers to this phenomenon as *halo effect*, a cognitive bias discovered from social psychology experiments (Beckwith et al., 1978).

2.2 Advertising IO: diffusion and pricing

Advertising industry consists in the production and diffusion of a good's media complement. The first task is ensured by advertising agencies whose role – as narrated in the *Mad Men* series – is to design the advertisement and its media plan. The trademark law internalizes the externalities associated with the production of advertising complement by granting firms with *brands* (which catalyzes ad effects). For its part, the diffusion of the complement is operated by two distinct ecosystems.

[1] It is worth noting Apple Macintosh's 1984 myth refers to Orwell's eponymous one.

On the one hand, offline (or traditional) advertising is a mass-media market where advertisers can only target a *context*: firms interested in sending their ads to fishermen can invest in fishing magazines' ad spaces (Evans, 2009). As a consequence, the offline ad ecosystem revolves around the notion of *audience* which has become the market's currency. Due to its strategic nature, audience measurement is provided by third-party firms, independent from advertising market players.

On the other hand, the online advertising market is characterized by low *targeting* costs (Goldfarb, 2014). The ad-selling activity – operated by a tangle of platforms (Estrada-Jiménez et al., 2019) – is integrated by dominant online media firms like Google and Facebook, a duopoly accounting for 51% of industry's revenues in 2019 (eMarketer, 2019). Ad sellers observe[2] and report the performance of their own advertisements: attention metrics, clicks, conversions on which they charge advertisers in pay-per-action contracts. The existence of such pricing models reveals another fundamental function of online advertisement: distribution. Indeed, an online ad is always "one click away" from the advertised commodity: hyperlinks provide a direct access to the good.

This is a qualitative leap in the nature of advertising. Advertisement is traditionally defined as a communication mean, semantic in nature (following Becker & Murphy) rather than a mean of access. Offline, advertising and distribution activities are separated. Their externalities are internalized by non-tariff vertical restraints between advertisers and distributors such as exclusive and selective distribution. By distributing L'Oréal, Carrefour benefits from the brand's adstock to attract clients and eventually sells his own private label goods. By imposing conditions on a good's access, distribution contracts internalize such external effects. However, they cannot be transposed online where advertising and distribution are operated by the same firms, leaving externalities uncompensated.

2.3 Externalities between advertising ecosystems

The existence of two distribution channels for advertisement has raised question about their competition and substitutability (Bergemann and Bonatti, 2011). The externalities between both media is, however, less considered. Yet, advertising is above all source of externalities: as a semantic object its effects are necessary "crossed" with other contextual elements of consumption such as seasonality, bandwagon effects, distribution or association with other media contents. The latter case is of interest here. In particular, the meaning produced by an ad complement is likely to be a good or a bad for other ads (Yan and Cruces, 2012). We refer to this phenomenon as *cross-media* externalities.

Such effects are empirically reported in the advertising literature under the concept of *media synergy*. Naik and Peters (2009) provide a survey of such effects and empirically demonstrate offline and online ads benefit from each other in building consumer's utility for a good. In a similar fashion, Kireyev et al. (2016) modeled externalities in-between online ads, investigating how display and search ads impact each others. It has also be found that TV advertisement generates a significant peak of Google and Yahoo! queries for a

[2] Through consumer's tracking technologies.

brand or product (Zigmond and Stipp, 2010; Lewis and Reiley, 2013). Another part of the literature, more theoretical, models externalities in-between online ads (Ghosh and Mahdian, 2008; Ghosh and Sayedi, 2010).

It would be better to talk about *externalities* rather than *synergies* since such effects are not internalized. Indeed, a super-bowl TV ad will not be compensated for rising an advertiser's Google queries, clicks and conversions on which it will be charged. The external effect may be either internalized by firm's cost function or by consumer's through a rise in retail prices. Cross-media effects are a type of *pecuniary* externalities *internalized* by online ad's price-per-action mechanism at the cost of advertisers, other media or consumers.

The *capture* of externalities is a common strategy, often encountered in digital economics. For example, telecom incumbent captured the network externalities of mobile usage by charging the access to local loop. In the same way, Google benefits from online services' network effects by guaranteeing access to internet services. In our case, we are not concerned with network but adstock externalities potentially internalized by digital ad sellers. These platforms act as *gatekeepers*, i.e. essential facilities in the access to goods and information. As such, they hold a right – one would say a monopoly – on their adstock. One could argue those effects exist offline. For example, shopping mall owners benefit from brands' media expenditures. However, in the physical distribution IO, the contractual chain between firms, retailers and shopping malls internalizes those effects while online such contracts do not exist. Moreover, while a shopping mall has a *local* market power, dominant online media are *global* gatekeepers of the internet.

To the best of our knowledge, the theoretical and empirical analysis of cross-media effects have been limited to a narrow range of media. Moreover, while such effects are often studied with the view of generating sales, they result in managerial rather than policy implications. In the following section, we propose an empirically modeling of cross-externalities between a set of both offline and online media. We drop the sales data to focus more precisely on how a media impacts consumers utility for online ads, as measured by the clicks it received. Such a phenomenon has important implications which are discussed later.

3 Empirical analysis

3.1 Data

We use advertising data from three brands $b \in \{A, B, C\}$. A and B are two brands belonging to the same German hotel group. The third one, C, is a British perfumer. The three dataset provide data reported on a weekly basis $t \in \{1, ..., n\}$. For brands A and B, our dataset provides the number of clicks on Facebook and Google ads. It also contains media spending in euro on several media, online as well as offline. Both brands invest on OOH[3], search, social and display advertisements. We also have data on advertisements for multiple brands of the holding, i.e. "multi-branding". The two brands also exhibit specific media-mix: brand A advertises on TV and print (i.e. press) advertising whereas B invests on radio and cinema. We

[3]Stands for Out-Of-Home advertisement, i.e. public display ads

also have other interesting features like the retail price of the advertised good, aggregated ad speeding from competitors and impressions on Google and Facebook ads. Dataset for brand C provides very similar data but with a different media mix. The firm invests on mail, e-mail, OOH, social, display and search ads. We observe the following consumers' responses: number of queries related to the brand, clicks on Google search ads and clicks on Google Shopping ads. Descriptive statistics are reported below in Table 1.

	Brand A		Brand B		Brand C	
Statistic	Mean	St. Dev.	Mean	St. Dev.	Mean	St. Dev.
#Google Clicks	15 885	6 493	44 979	13 971	39 011	26,148
#Google impressions	75 171	35 454	161 281	63 867	558 923	345 398
#Facebook Clicks	661	1 354	1 878	4 514		
#Facebook impressions	231 618	470 656	754 497	1 884 239		
#Google shop clicks					7 798	4 802
#Google shop imp					450 022	302 053
#Google queries					478	299
Price (EUR)	112	7	75	6		
Social (EUR)	550	977	1 517	4 097	386 177	477 250
Display (EUR)	4 070	5 876	11 047	33 485	215	451
OOH (EUR)	1 265	5 671	3 314	9 657	2 393	18 288
Multi-brand offline (EUR)	18 384	82 076	12 714	22 12		
Multi-brand Online (EUR)	2 372	21 657	2 372	21 657		
Competitors (EUR)	475 099	501 766	475 099	501 766		
Radio (EUR)			71	916		
Cinema (EUR)			3 035	32 670		
TV (EUR)	3 271	23 258				
Print (EUR)	258	2 595			3695	27039
n	190				118	
Period covered	01/2016 to 07/2019				06/2016 to 09/2018	

Table 1: Summary statistics, (EUR) indicates a monetary value and # a count.

High standard deviations are common in advertising where both media investments and consumers' behaviors exhibit strong seasonality, as we can see in Figure 1 and 2. What the top plots make clear is that offline and online budgets are distributed differently over time. Offline, firms only advertise during precise periods (e.g. Easter, Christmas). Whereas online, they invest all along the year and their investments are less sensitive to season peaks.

The bottom plots of Figure 1 and 2 highlight the different volatility of consumer's responses. In particular, they catalyze the technical complexity of cross-media effects measurement. Ad spending and consumers behavior exhibit the same seasonality, making difficult to infer a causal relationship of the first on the second. Does advertising on one media generates clicks on another one? Or does consumer seasonal interest for the good trigger firm's multi-media investments? The next subsection provides our identification strategy.

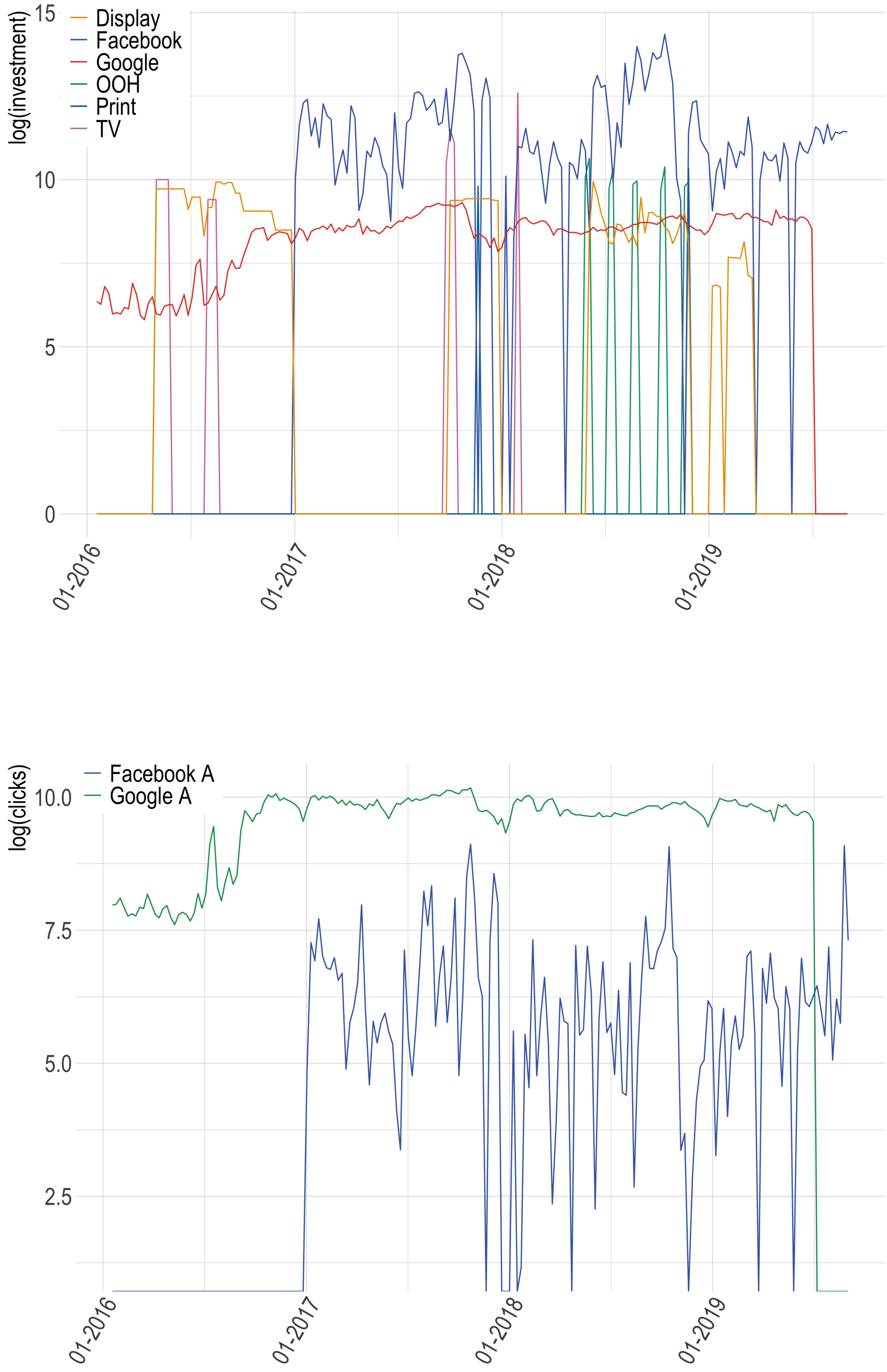
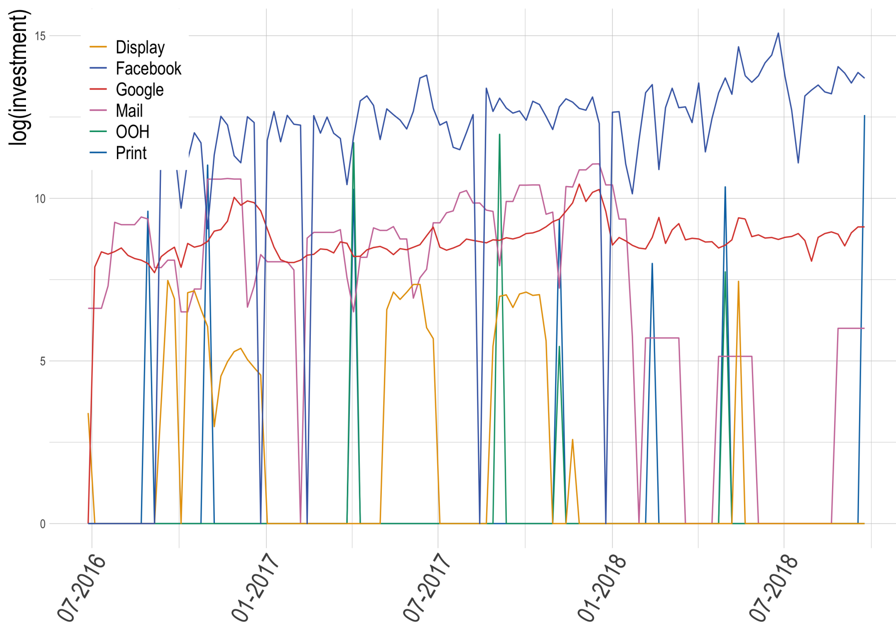


Figure 1: Evolution of brand A's media investment (top) and clicks on online ads (bottom) through time.



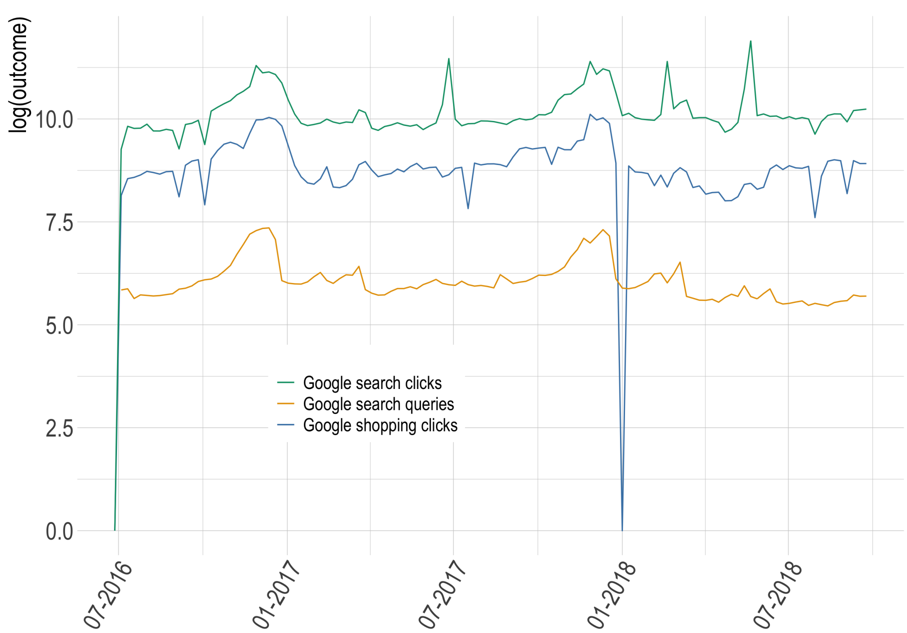


Figure 2: C's media spending (top) and Google clicks (bottom) through time.

3.2 Modeling approach

We seek to estimate how the effectiveness of Google and Facebook ads may be impacted by externalities from other media. Each of our three advertisers invests on a set of media indexed by $m \in \mathcal{M}$. Among this set, let $i \in \{G, F\}$ index Google and Facebook, the subset of media we are interested to model.

Dependent variables We approximate the effectiveness of media i in t by the total number of clicks recorded U_{it}. Clicks serve as a proxy for consumer's utility for the ad. It is also a pricing instruments on which advertisers are charged in pay-per-clicks contracts. We model clicks received on three media i: (i) Google search ads for all three brands, (ii) Facebook ads [4] for brand A and B and (iii) Google Shopping ads for brand C. We also model C's weekly number of queries as a proxy for brand's online notoriety.

Independent variable We seek to explain online media effectiveness – as measured by three outcomes described above – by the level of different media investments. As discussed in 2.1, advertising's effect is inter-temporal. Thus, we use adstock to (imperfectly) account for the cumulative effect of advertising. For each media m, we compute the following adstock A_{mt} of the period t:

$$A_{mt} = \sum_{\ell=0}^{t} \delta_m^{t-\ell} a_{m\ell}, \tag{1}$$

where $a_{m\ell}$ is the media investment (in EUR) in week $t = \ell$ and δ_m a media-specific carryover rate. Following the literature, we set TV's carryover-rate $\delta_{TV} = 0.9$ (Dubé et al., 2005; Dubois et al., 2017; Shapiro et al., 2020). The other rates are then set based on assumptions and empirical expertise of the authors, all reported in table 2.

Media m	δ_m
TV & Cinema	0.9
Radio & Cinema	0.7
OOH & Print	0.5
Direct mail	0.4
Display & Social	0.4
eMail	0.2
Search link	0.15
Competitors	0.5

Table 2: Adstock parameters by media, they are the same across the three brands. The key assumption behind those δ_m is that the adstock increases with the media richness of an ad (e.g. image, sound, both of them) and its audience. The last assumption relies on the network effect phenomenon underlying media economics.

[4] Facebook data are reported on heterogeneous time period. We provide their calculation in appendix.

3.3 Identification strategy

The specificity of our dependant feature is to be a count data since $U_{it} \in [\![0, +\infty]\!]$. We take into account this specificity by specifying the following regression for our Google clicks:

$$\ln(U_{it}) = \alpha_i + \beta_i \ln(\mathbf{I}_t) + \sum_{m \in \mathcal{M}} \gamma_{im} \ln(1 + A_{m,t-1}) + \theta_i \mathbf{X}_t + \tau_i \mathbf{S}_{m(t)} + \lambda_i t + \varepsilon_{it}. \quad (2)$$

In equation (2), our dependent variable is drawn on a Negative Binomial distribution. This specification accounts for the over-dispersion suggested in Table 1 and further documented in appendix. α is a media intercept and ε an error term. We estimate the effectiveness of advertising media i in period t as a function of several terms. $\mathbf{I}_t$ is a vector of in-period aggregated ad spending ; $A_{m,t-1}$ is the media m's past advertising stock ; $\mathbf{S}_{m(t)}$ a vector containing 11 month dummies ; t is a trend feature and $\mathbf{X}_t$ a vector of controls: retail prices for each brand and impressions recorded on media i.

The economic intuition of this model that *ceteris paribus* advertising is a Good or a Bad, complementary to online media i. From the regression coefficient of interest γ, we can derive the following elasticity:

$$\exp(\gamma_{im}) - 1 = \frac{\partial U_i}{\partial A_m} \frac{U_i}{A_m}$$

To be verified, our assumption implies that we control for other common effects between advertising expenditures and consumer's responses. First, β isolates the immediate effect of ad spending on clicks. However, this relationship is likely to be endogenous since – although we use several controls – it is not always clear whether ad investments trigger consumer's behaviors or the opposite. Indeed, as Shapiro et al. (2020) pointed out: firms may advertise on period during which they expect higher advertising effectiveness (e.g. Christmas).

This endogeneity concern is the reason why we implement a one week lag in our explanatory variable of interest, $A_{m,t-1}$. Vector θ controls for the variation in consumer prices and advertising audience (number of impressions recorded) which may have its own seasonality. Finally, τ and λ captures trend and seasonal effects such as Valentine's day, Oktoberfest or Christmas where ad investments and clicks commonly rises. By controlling for all of these features, the externalities measured by γ will be time, audience and price invariant.

3.4 Robustness check

We estimate equation (2) by Maximum-Likelihood estimation. To handle the risk of autocorrelation with omitted variables, we implement Newey and West (1987)'s robust standard-errors. We use McFadden's adjusted pseudo R^2 to evaluate the relative performance of our models:

$$\text{Pseudo } \mathrm{R}^2 = 1 - \frac{D_\Theta + K}{D_\alpha} \quad \in [0, 1]$$

This criterion can be interpreted as the improvement in the deviance D allowed by our set of parameters $\Theta = \{\alpha, \beta, \gamma, \lambda, \theta, \tau\}$, compared to a null model with a standalone intercept α. This pseudo R^2 takes in account the number of parameters K, penalizing models with too many features. It is thus a useful metric to measure the relative performance of our model following several robustness check performed in the following paragraphs.

Indeed, the choice of adstock rates δ_m and lag parameter $(t-1)$ can be the subject of legitimate criticism of our method. We provide evidence that our models are robust to changes in adstock and lag parameters.

Adstock rate Regarding the first issue, we run the model presented in equation (2) by specifying adstock variables with an "inverted" carryover $\delta_m^{'} = 1 - \delta_m \;\; \forall m \in \mathcal{M}$. We then reported the variation in Pseudo R^2 following this change in table 3. The overall significance of our model only vary marginally with this new specification.

Lag parameter Regarding the lag parameter chosen to avoid endogeneity, we run alternative regressions of (2) for brand A, where adstock variables are lagged by $\tau \in [\![2, 4]\!]$ weeks. Such models investigate mid-term effects of adstock on clicks. The results reported in appendix (table 7) show similar coefficients.

	Brand A		Brand B		Brand C	
	Google	Facebook	Google	Facebook	Google	GS
$\delta_m^{'}$	0	0.05	-0.01	-0.02	0.01	-0.02
$\tau \in [\![2, 4]\!]$	0	0	–	–	–	–

Table 3: Models robustness to adstock rate setting. Value reports the change in Pseudo-R^2 following a change in adstock rate from δ_m to $\delta_m^{'}$ and from A_{t-1} to $A_{t-\tau}$
.

Table 4: Regression results for equation (2)

	Brand A		Brand B		Brand C		
	Google	Facebook	Google	Facebook	Queries	Search	Shopping
Google	0.135^{***}	-0.209^{**}	0.066^{***}	0.057	0.042	0.100	0.084
	(0.046)	(0.023)	(0.092)	(0.053)	(0.026)	(0.071)	(0.083)
Facebook	0.011^{*}	-0.016	0.010^{*}	0.025	0.008^{***}	-0.005	0.016^{**}
	(0.006)	(0.158)	(0.006)	(0.091)	(0.002)	(0.007)	(0.006)
Display	0.002	0.143^{***}	0.002	-0.041	0.006^{*}	0.007	0.014^{*}
	(0.006)	(0.040)	(0.005)	(0.036)	(0.004)	(0.010)	(0.009)
OOH	-0.006	-0.176^{***}	0.008	0.161^{***}	-0.003	-0.013^{**}	0.025^{***}
	(0.005)	(0.036)	(0.005)	(0.046)	(0.002)	(0.005)	(0.007)
Comp.	-0.051^{***}	0.142	-0.039^{***}	-0.047	-0.046^{***}	-0.012	0.026
	(0.019)	(0.153)	(0.013)	(0.136)	(0.011)	(0.026)	(0.041)
TV	0.022^{***}	-0.021					
	(0.007)	(0.053)					
Print	0.009	0.101			0.007^{***}	0.013^{**}	-0.015^{***}
	(0.006)	(0.081)			(0.002)	(0.006)	(0.005)
Cinema			0.003	-0.061^{**}			
			(0.004)	(0.026)			
Radio			0.010^{*}	-0.070			
			(0.006)	(0.076)			
Mail					0.041^{***}	-0.022^{**}	-0.007
					(0.003)	(0.011)	(0.013)
eMail					0.037^{***}	0.059^{**}	-0.024
					(0.009)	(0.029)	(0.031)
Controls							
Trend	X	X	X	X	X	X	X
Months	X	X	X	X	X	X	X
Impressions	X	X	X	X		X	X
Price	X	X	X	X			
Pseudo-R^2	0.94	0.45	0.92	0.55	0.88	0.90	0.89

Note: $^{*}p<0.1$; $^{**}p<0.05$; $^{***}p<0.01$

4 Results and discussion

Tables 4 provides the results of our model for all three brands. For readability, we only displayed the γ_{im}, our coefficients of interest. Other coefficients are reported in appendix (table 6) whereas controls variables are indicated for each regressions. The specification of our model allows to interpret our coefficients of interest γ_{im} as a cross-media elasticity, i.e. a $\{100 \cdot [\exp(\gamma_{im}) - 1]\}\%$ *variation in the outcome of media i after a 1% increase in m's past advertising stock.*

Our results suggest the existence of externalities generated by online and offline ads on online media effectiveness, on all three brands. Cross-media effects are very heterogeneous across brands and media. Indeed, it is in the nature of advertising to generate a variety of effects among consumers, since the semantic nature of the good makes its utility heterogeneous. Nevertheless, we can identify some trends in our results.

Offline, cross-media effects are mainly positive: Google seems to benefit from positive effects from TV, print and radio advertisements. TV boosts Google clicks by 2.4%, Radio by 1% and print by 1.3%.

Externalities between Google and Facebook ads are also interesting. On the three brands Google seems to benefit from Facebook ads. Looking closer at brand A, Google even cannibalizes Facebook clicks. Facebook neither benefits from nor cannibalizes Google ads. Theses coefficients may illustrate a competition for clicks between the so-called Google-Facebook advertising duopoly. Indeed, other online ads such as display and eMail positively impacts Facebook and Google's clicks.

OOH on its side exhibits ambiguous effects: its effect on clicks tends to be negative. Theoretically, this may be a sign that outdoor campaigns are perceived as a Bad by consumers. Another and more practical rational for this effect be related to a correlation error since we cannot control for the location of ads which is, by definition, very important for outdoor campaigns.

From a theoretical point of view, we can empirically observe that an ad on a given media is a Good or a Bad – depending on $\mathrm{sgn}(\gamma)$ – for ads on other media. The negative signs demonstrate the idea that consumers may dislike a given advertisement and thus may be less willing to click on other ads online. In the words of Becker and Murphy (1988) on rational addiction, we can say advertising exhibits *adjacent* complementarity ($\gamma > 0$) or substitution ($\gamma < 0$) since utility for ads in t depends on consumer's previous ads consumption stock.

This empirical study proposes a quantification of externalities between advertising media. Our model provides significant results. However, it suffers from both technical and theoretical limitations which must be mentioned.

On the modeling itself, the results for brand C must be taken carefully. Indeed for brand C, the data on prices is not available, weakening the robustness of our results. Another main technical limitation of our model is the number of observations. This limitation is common in advertising, where data are reported on a weekly basis. Although, it covers three years our dataset has a small size, making it sensitive to *measurement noise* (Naik et al., 2007). This is also a limitation to the complexity of our models, since we have low degrees-of-freedom. Using interaction terms between media and seasons or polynomial function forms would have been interesting but in our case, it would have consumed too many degrees-of-freedom.

More broadly, it is not possible to know which consumer has consumed which campaigns since consumer-level data are not observable offline. Also, and as mentioned previously, advertising returns are neither monotonous nor immediate. They highly depend on semantic characteristic specific to the advertising message (e.g. information, narratives) and the context around it (e.g. bandwagon effect, media coverage). By favouring a quantitative analysis, we aggregated weekly budgets and clicks and thus we disregarded the semantic characteristics of each campaigns: e.g. format, message, population targeted. In short, the present paper only focuses on a certain measure of a certain type of advertising externalities. The presence of such complex effects is all the challenge of advertising research.

5 Implications

5.1 Managerial implications for advertisers

The presence of externalities between advertising media has important implications for firms' media investment strategies. Indeed, our results suggest that each time an advertisers invest on a media, it increases the effectiveness, and thus the cost, of online media. To have an approximation of such effects, we convert our elasticities γ_{im} in values with the following back-of-the-envelope equation:

$$\frac{d\text{Cost}_i}{dA_m} = \underbrace{\left([\exp(\gamma_{im}) - 1] \cdot \bar{u}_i\right)}_{\eta_{im}} \cdot \bar{c}_i \equiv \xi_{im}, \tag{3}$$

where $\bar{u}_i$ and $\bar{c}_i$ respectively denote the average clicks and cost-per-clicks. Equation 3 provides a coefficient η_{im} which can be understood as the variation in the number of clicks on media i following a 1% increase of media m's adstock value. The variation of clicks logically results in a cost variation of ξ_{im}€. We report some Google search's clicks and cost elasticities to in table 5.

	Brand A		Brand B		Brand C	
	η	ξ	η	ξ	η	ξ
TV	364	112€	–	–	–	–
	(355€)					
Radio	–	–	212	45€	–	–
			(30€)			
Print	non-sign.		–	–	436	128€
					(72€)	
Compet.	−762	−233€	−1651	−363€	non-sign.	
	(9000€)		(9000€)			

Table 5: Google's clicks and cost elasticities to other media investments. E.g. investing 335€on TV ensures brand A 364 weekly additional, generating an additional cost of 112€.

Indeed, as table 5 highlights, advertising a surplus of ΔA_m euros on media m occurs a ξ_{im} euros increase in online ad cost.

To date, advertisers enlighten their investment decisions with Media Mix Modeling (Chan and Perry, 2017) or Multi-Touch-Attribution (Abhishek et al., 2012). These methods leverage respectively regression and Markov chain analyses to provide an *ex-post* evaluation of ad investments, know in the literature as *attribution* modeling. Externalities between media represent an additional challenge for such models. As discussed earlier, such effects have already been highlighted in the literature. This study provides further evidence of cross-media effects by considering a wide range of media and offline-online interactions in building precise online outcomess (clicks and queries).

In particular, our study is in line with a preoccupation of marketer which Chan & Perry called *funnel effects*, i.e. the impact each media may have along the consumer journey, from awareness to (re)purchase or churn. This challenge is made more difficult by the asymmetry between ads data available online and offline.

Whether clicks effectively lead to sales and is thus profitable is another question. In their study, Blake et al. (2015) find a low effectiveness of keyword-based brand advertisement. We support the hypotheses that firms still advertise on their keyword and audience because of their opportunity cost of letting such strategic places to competitors. We will document further this idea later.

5.2 Advertising media substitutability

The corollary implication for media firms consists in internalizing cross-media effects as well as asserting their economic function. In other words, this study questions the idea that online and offline advertising belong to the same relevant markets and are thus substituable (Goldfarb and Tucker, 2011b,a). Since online media effectiveness seems dependant of traditional investments, both ad markets may be as much complementary as they are substitutes – like He et al. (2018) highlighted.

Indeed, the difficulty of the complementary-or-substitute debate lies in the ambiguous function of online advertisement which endows goods with a (i) media representation but also with (ii) an access link. Bomsel et al. (2013) pointed out early that these two economic functions generate distinct utilities. While every advertising message generates a *communication* (dis)utility, as stated by Becker and Murphy (1993), online ads also provides consumers with an *access* utility. It is worth noting that this access utility is strongly dependent from consumer's utility for the good which is built by other media investments, especially offline ones which role is to provide a *communication* utility. The descriptive statistics displayed earlier in figure 1 and 2 show that firms' investment patterns online are more similar to distribution investments sustained all the year than the usual seasonal media buying strategies.

We argue that – along with targeting – hyperlink is a major economic difference between online and offline advertising. Hyperlink and consumer's tracking opened the path to new advertising pricing models which change the economic nature of advertisement by endowing advertising with a *distribution* function. In cost-per-clicks models, firms purchase clients in their online store. In cost-per-acquisition, the ad seller is associated to each sales generated by the firm[5].

Thus, the extent to which offline and online ad industries belong to the same unique relevant market of "advertising" is questionable. Because it ensures a distribution function, online advertising should be regarded, at least partly, as a proper relevant market, distinct from the offline one. Reconsidering online advertising as point-of-sale advertisement or distribution mean would redefine its regulation, pricing and measurement. The market definition is crucial since offline media – uncompensated for their effects on online media they are competing with for advertiser's budget – may suffer from the current common relevant market. The question to know if traditional media advertising is sub-priced and the market definition call for further antitrust and competition policy analysis.

For the moment, our study echoes the very topical debate on revenues sharing between traditional and online media.

5.3 Regulating online advertising media

Cross-media effects belong to the family of *pecuniary* externalities. Thus, they do not denote a market failure *per se*. On the contrary, they may be the sign of a competitive markets between media for advertiser's revenues (Liebowitz and Margolis, 2002). However, those externalities may be part of regultor's concerns since they are captured by integrated media-ad sellers with high market power.

A clear example of such concerns may be found in the French and Australian hot debate around press' revenues sharing. France has been the first country to apply the 2019's EU *copyright* directive, allowing media to control and remunerate the online diffusion of their contents, notably on Google and Facebook. Australia has been a more recent battlefield around press revenues sharing. While Google accepted Canberra's condition on press content remuneration, Facebook temporarily deleted articles from its platforms before finding an agreement with public authorities (FT, 2021).

This stories demonstrate that Google and Facebook act as a *symphon* which drain newspaper's audience and advertising revenues. The same mechanism is at stake on the advertising side of the media activity. Through their pricing and effectiveness indicators (e.g. clicks), online media capture online effects of traditional advertising, possibly at the expense of firms, consumers and offline media.

Thus, cross-media effects may be a concern for regulators. And particularly in a context where Google and Facebook are already probed with abuse of dominance: self-reporting of advertising effectiveness, restriction of ad inventory selling[6] (Competition & Markets Authority, 2020).

This study is concerned with the fact that digital advertising media are in a position of *gatekeepers* regarding the information and distribution of goods online. Newspaper and law cases provide some illustration of this. Indeed, one year ago, a noodle shop owner complaints Google first displayed delivery services when

[5] Such contracts implement new agent-principles problems in the advertiser-media relationship described in Hu et al. (2016)

[6] Youtube.com ad spaces are sold through Google's platforms and Facebook.com ones are distributed *via* Facebook Ads

consumers searched him (NYT, 2019). Earlier, Interflora suited its competitor Florajet for buying the ad slot on "Interflora" Google results (DLAPiper, 2015).

In short, Google seems to allow any advertiser to free-ride a brand's notoriety by selling an ad slot on its search results. This could lead to a rise in price and/or demand for notorious brands' keywords. The welfare implications of such a phenomenon depend on (i) whether firms and traditional media significantly suffer from online ads pricing schemes and (ii) the extent to which they defer this loss on consumer prices.

6 Conclusion

Our study aims to provide a novel framework on the debate and research on advertising media effectiveness, substitutability and regulation. The originality of this study is to consider advertising as a good which provides consumers with a meaning about the good. We then question whether ad sellers may capture the semantic value of other advertisements for their benefits. The empirical analysis provided seems to confirm the existence of such effects. We found significant and mostly positive impact of different media on online ads effectiveness. Our findings are related to the recent trend which questions the real effectiveness of online ads among which the recent essay of Hwang (2020). If the author criticizes the opacity of advertising targeting, our study rather highlights the economic mechanism by which online ads benefit from offline ones.

As we discussed, those cross-media effects are internalized by online advertising media's pricing models. Yet, these digital media firms, endowed with high market power, ensure both advertising and distribution of goods. They seem to capture externalities from other advertising media at the expense of advertisers, traditional media and competition notably because, thanks to their gatekeeping position, they can tap the flow of consumers generated by offline ads. Moreover, the capturing scheme are very varied: Facebook's capture strategy may differ from the Google's one. Both would deserve further analysis.

The technical and theoretical limitations of this study opens the path to further modeling and policy topics. In particular, the study may be replicated on a more diverse set of time and industries in order to detect patterns in cross-media effects. Though our results are globally robust and interpretable, more sophisticated specification could be applied, both in adstock and media effectiveness modeling. Managing seasonality while leveraging more *causal* method than regression analysis are two important rooms for improvement. Finally, the optimality and relevance of suggested policy implications deserves specific analyses. All these ideas are as much future research directions possible to extend the topic of cross-media externalities. What is certain is that such regulations would dramatically redefine the digital economy which has always been relying on advertising.

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7 Appendix

7.1 Additional descriptive statistics

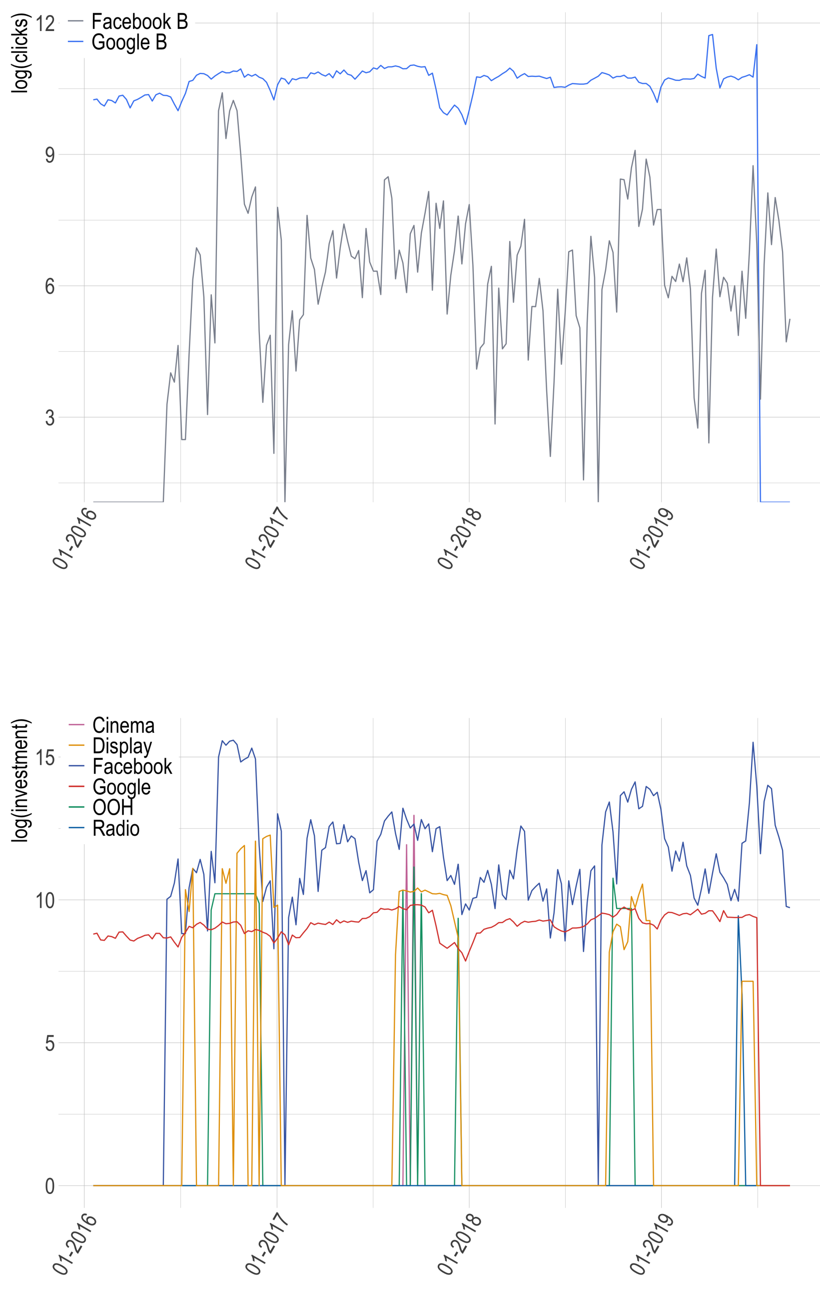

Figure 3: Trends in brand B's clicks (top) and media mix (bottom)

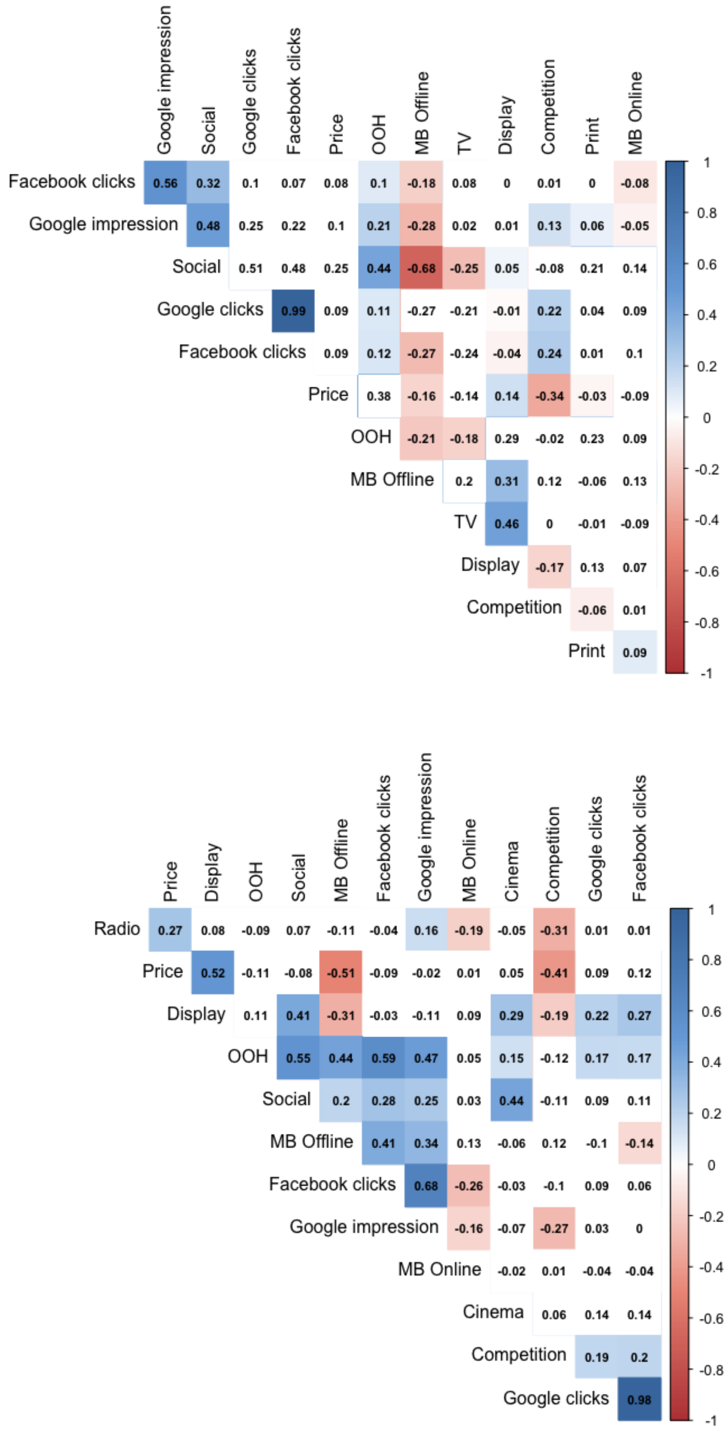

Figure 4: Correlation matrix for brand A (top) and B (bottom) features

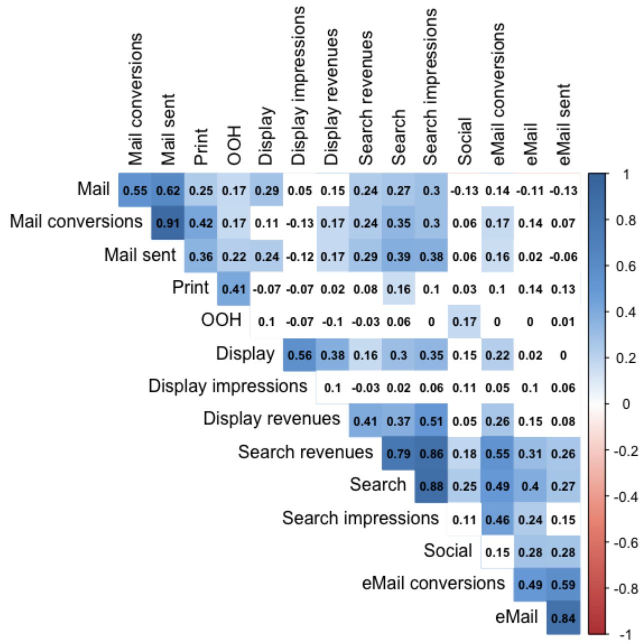

Figure 5: Correlation matrix for brand C features

7.2 Facebook weekly data computation

The heterogeneous lengths of social ad campaigns is an issue we have to deal with since all our explanatory features are reported on calendar weeks. Let $U_{f,p}$, $\mathrm{Imp}_{f,p}$ and $Cost_{f,p}$ respectively be the clicks, impressions and cost associated with Facebook ad campaigns for each period p on a $d(p)$-day length period. We convert data from heterogeneous period length into weekly period as following:

$$\begin{aligned} U_{f,t} &= \sum_{p \in \mathcal{P}} \big\{[U_p/d(p)] \cdot d(p \cap t)\big\}, \\ \mathrm{Imp}_{f,t} &= \sum_{p \in \mathcal{P}} \big\{[\mathrm{Imp}_p/d(p)] \cdot d(p \cap t)\big\}, \\ \mathrm{Cost}_{f,t} &= \sum_{p \in \mathcal{P}} \big\{[\mathrm{Cost}_p/d(p)] \cdot d(p \cap t)\big\}. \end{aligned} \tag{4}$$

The terms inside parentheses correspond to the daily-average Facebook features in week t. Period p overlaps week t on a number of days $0 \leq d(p \cap t) \leq 7$.

7.3 Additional regression results

Table 6: Regression results for other covariates.

	Brand A		Brand B		Brand C		
	Google	Facebook	Google	Facebook	Queries	Search	Shopping
Online_t	-0.022^{*}	-0.453^{***}	0.017^{**}	-0.339^{***}	0.077^{***}	-0.014	-0.001
	(0.013)	(0.089)	(0.009)	(0.063)	(0.007)	(0.025)	(0.017)
Offline_t	-0.009^{**}	-0.046	0.0004	-0.015	0.009^{***}	0.016^{**}	0.001
	(0.004)	(0.034)	(0.003)	(0.015)	(0.002)	(0.007)	(0.006)
Competitors_t	0.003	-0.014	0.020^{**}	-0.058	-0.007	0.039	-0.072
	(0.013)	(0.078)	(0.008)	(0.074)	(0.019)	(0.039)	(0.055)
t	0	0	0	0	0^{***}	0	-0
	(0.001)	(0.001)	(0.001)	(0.001)	(0)	(0.001)	(0.001)
α	8.055^{***}	11.604	10.972^{***}	-5.260	12.457^{***}	7.107^{**}	14.823^{***}
	(2.440)	(16.983)	(1.300)	(13.783)	(0.848)	(3.169)	(4.207)
Price_t	0.003	0.012	0.004	-0.067			
	(0.002)	(0.019)	(0.005)	(0.046)			
Imp_t	0^{***}	0^{***}	0^{***}	0^{***}		0^{***}	0^{***}
	(0)	(0)	(0)	(0)		(0)	(0)
eMail_t					0.044^{***}	-0.008	-0.005
					(0.006)	(0.017)	(0.017)
Pseudo-R^2	0.94	0.45	0.92	0.55	0.90	0.89	0.88
Note:							*p<0.1; **p<0.05; ***p<0.01

7.4 Robustness checks

7.4.1 Lag parameter

Table 7: Regression results for brand A testing for several $A_{t-\tau}$

	$\tau = 2$		$\tau = 3$		$\tau = 4$	
	Google	Facebook	Google	Facebook	Google	Facebook
TV	0.024***	-0.021	0.026***	−0.021	0.029***	−0.021
	(0.006)	(0.05)	(0.006)	(0.05)	(0.006)	(0.05)
OOH	−0.007	-0.176***	−0.005	−0.176***	−0.036	−0.176***
	(0.005)	(0.003)	(0.004)	(0.03)	(0.04)	(0.004)
Print	0.01	0.101	0.013**	0.101	0.150**	0.101
	(0.007)	(0.08)	(0.006)	(0.08)	(0.006)	(0.08)
Google	0.103*	−0.209**	0.135**	−0.209*	0.160***	−0.209**
	(0.05)	(0.078)	(0.05)	(0.074)	(0.04)	(0.09)
Display	−0.002	0.1433***	0.001	0.143***	−0.001	0.143***
	(0.004)	(0.090)	(0.04)	(0.03)	(0.004)	(0.04)
Social	0.001	−0.016	0.007	−0.156	0.003	−0.016
	(0.006)	(0.158)	(0.006)	(0.160)	(0.005)	(0.160)
Competitors	−0.04*	0.142	−0.055**	0.142	−0.061***	0.142
	(0.024)	(0.15)	(0.02)	(0.15)	(0.015)	(0.153)
Pseudo-R^2	0.94	0.45	0.95	0.45	0.95	0.45

Note: *p<0.1; **p<0.05; ***p<0.01

7.4.2 Other specification

This section provides results obtained with alternative specifications. First, we estimate equation 2 by OLS, assuming $U_{it} \in [-\infty, +\infty]$ and thus not taking in account count data property.

We then estimate the model using a Poisson regression, drawing $U_{it} \sim \text{Poisson}(\lambda_{it})$. Here, we assume that mean and variance of clicks are equal. Thus, we do not account for over-dispersion.

Finally, taking advantage of the time series property of our data, we test the following model:

$$\ln(U_{it}) = \alpha_i + \beta_i t + \sum_m \gamma_{im} \ln(1 + A_{m,t-1}) + \sum_m \sum_{l=-1}^{1} \eta_{iml} \Delta \ln(1 + A_{m,t-1-l}) + \lambda_i \Delta \ln(\mathbf{I}_t) + \theta_i \mathbf{X}_t + \tau_i \mathbf{S}_{m(t)} + \varepsilon_{it}.$$

This Dynamic OLS specification takes in account a potential co-integration between clicks and ad spending series. By taking l leads and lags, the DOLS estimator accounts for possible simultaneity in the regression as well as long-term relationship between our main features. Last but not least, DOLS estimators have demonstrated good performances on small sample size (Stock and Watson, 1993).

Results of this benchmark are reported for brand A in the following table. It shows that OLS seem biased as estimates deviate from NegBin, Poisson and DOLS coefficients. As expected, Poisson estimates show very close results compared to NegBin ; coefficient are globally lower and more significant. Finally, DOLS results are very close from Poisson and NegBin models.

Table 8: Test for other specifications, all estimated with Newey-West's robust standard errors.

	OLS		Poisson		DOLS	
	Google	Facebook	Google	Facebook	Google	Facebook
TV	0.021***	−0.080***	0.019***	−0.034	0.018**	−0.02
	(0.007)	(0.028)	(0.006)	(0.055)	(0.007)	(0.06)
OOH	−0.005	−0.171***	−0.005	−0.112**	−0.004	−0.289***
	(0.005)	(0.028)	(0.004)	(0.054)	(0.004)	(0.003)
Print	0.009	0.015	0.010**	0.106	−0.015***	0.08
	(0.006)	(0.057)	(0.004)	(0.102)	(0.004)	(0.05)
Google	0.133***	−0.116***	0.169***	−0.210***	0.431***	−0.064
	(0.046)	(0.042)	(0.037)	(0.046)	(0.009)	(0.083)
Display	0.001	0.028	0.006	0.075**	−0.002	0.008
	(0.006)	(0.021)	(0.004)	(0.035)	(0.004)	(0.05)
Facebook	0.011*	0.033	0.010**	0.066	0.001	1.201***
	(0.006)	(0.050)	(0.004)	(0.204)	(0.005)	(0.081)
Competitors	−0.050***	0.126*	−0.052***	0.130	−0.037*	0.214
	(0.019)	(0.075)	(0.013)	(0.156)	(0.02)	(0.142)
R^2	0.95	0.60	0.95	0.69	0.97	0.56

Note: *p<0.1; **p<0.05; ***p<0.01