Systemic risk in energy derivative markets: a graph-theory analysis

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Objectives

- Empirical study on systemic risk in derivative markets
- Approach in three dimensions
  - Observation time
  - Spatial integration
  - Maturity of the transactions
- Influence of physical as well as derivative markets

- Integration as a necessary condition for systemic risk to appear
- Correlations, co-movements

Selected markets

- Choice directed by:

- Concerns about speculation in commodities

Energy products

- Development of bio fuels

Agricultural products

- Portfolio management / Commodities as a new class of assets

Financial instruments

- Organized markets with the highest transaction volumes

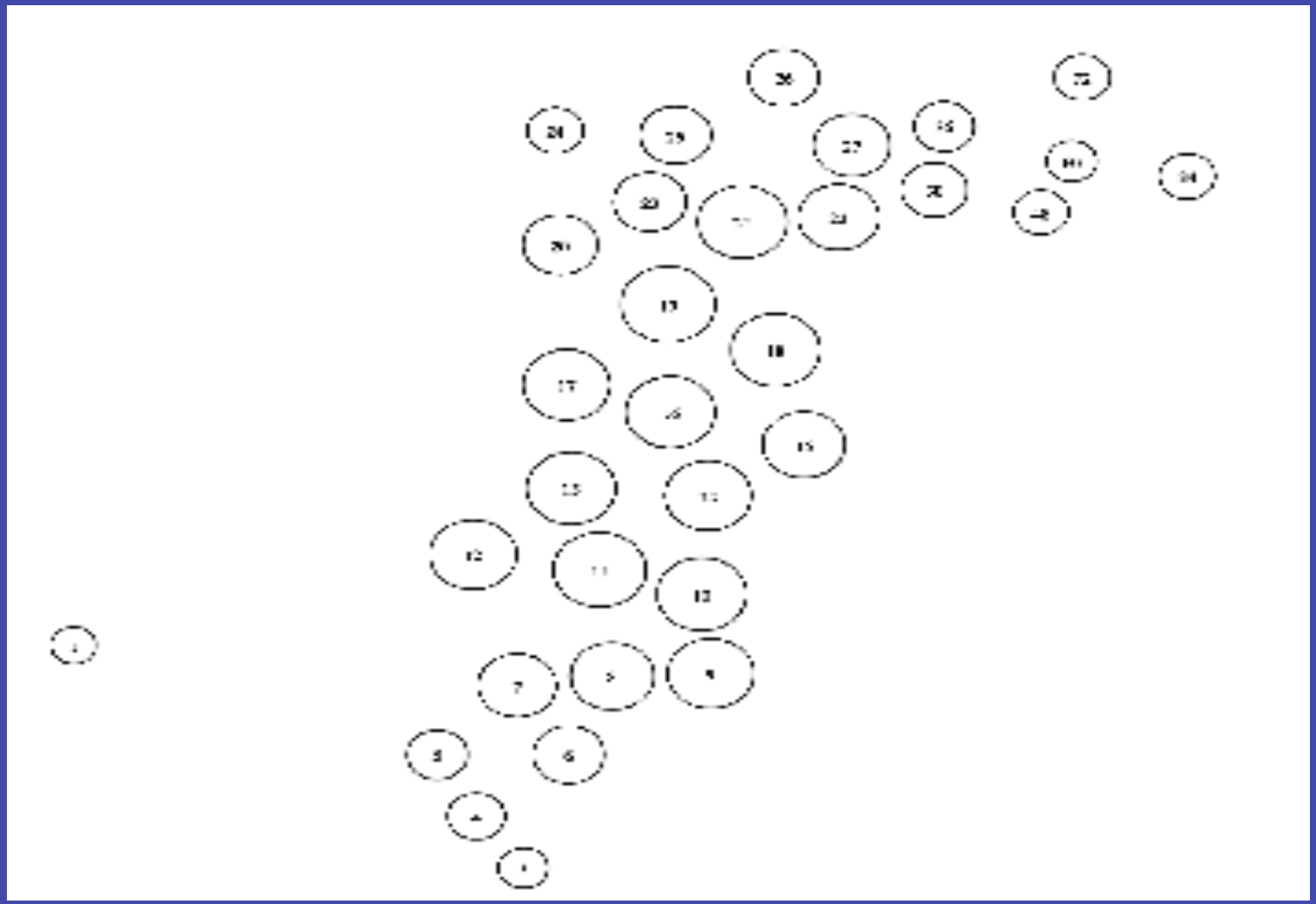
- 14 markets (> 760 000 daily futures prices (settlement))
- 1998 - 2011

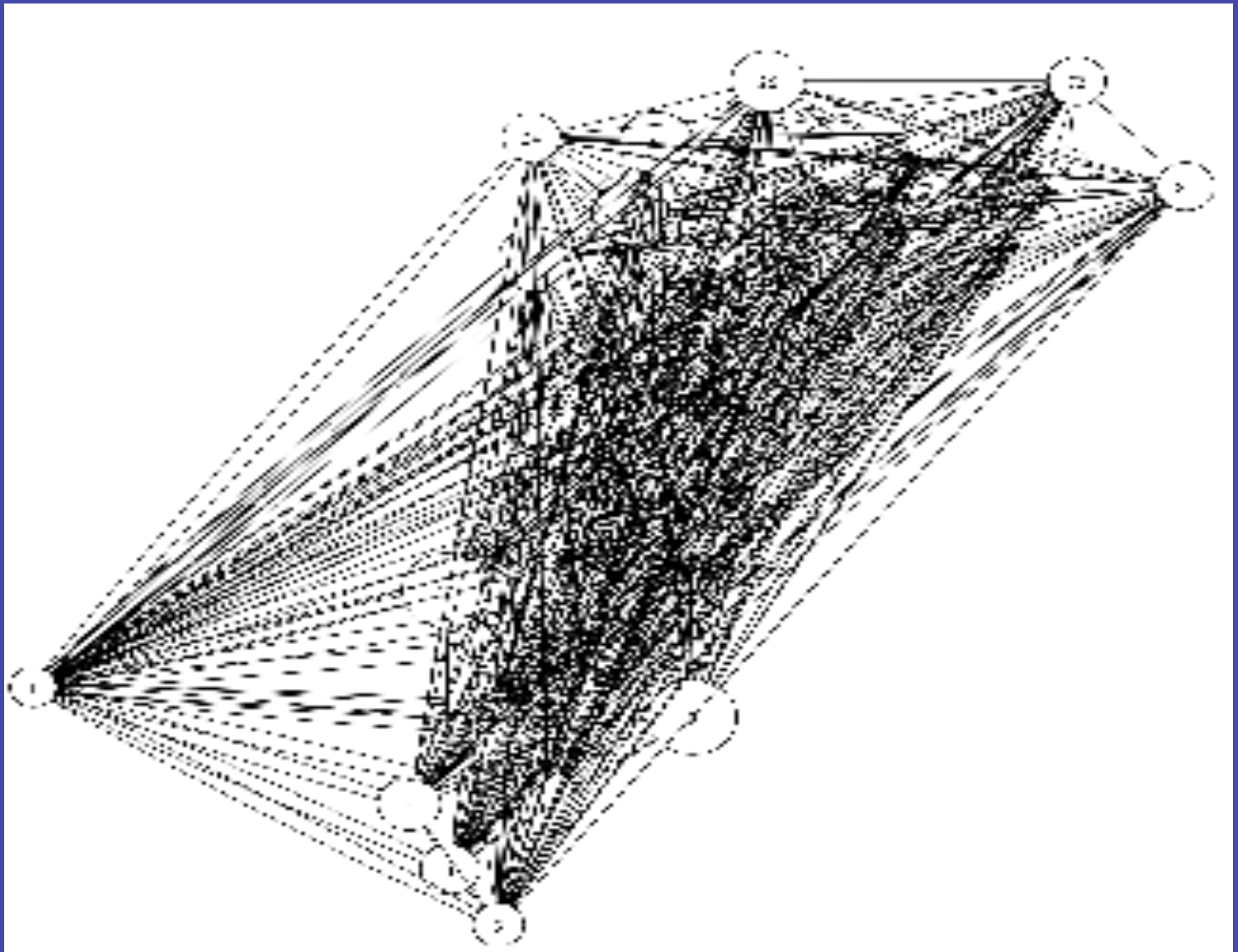
Methodology

- Huge volume of data + 3 dimensional analysis
- Price system = complex evolving system
- Use of methods originated from statistical physics
- Graph-theory and networks
- Graph : - Nodes : time series of daily futures returns
  - Links : correlations between the price returns
- Full connected graph :

  all possible connections between N nodes (time series of price returns) with (N(N-1)/2) links

Example of the crude oil market (1 market, maturity dimension)





Methodology

- Information in the graph is filtered
  - most relevant links
  - highest correlations
- Minimum Spanning Trees (MST)

Mantegna (1999)

Path for the propagation of prices fluctuations

1. How did we get minimum spanning trees?
2. Our results with the MST, in the 3 dimensions :

   maturity, space, 3D
3. Dynamical analysis on the graphs and on the MST

1. Minimum spanning trees

- Synchronous correlation coefficients ρ of prices returns r :

$$r_i = \frac{\left(\ln F_i(t) - \ln F_i(t-\Delta t)\right)}{\Delta t}$$

$$\rho_{ij}(t) = \frac{\langle r_i r_j \rangle - \langle r_i \rangle \langle r_j \rangle}{\sqrt{\left(\langle r_i^2 \rangle - \langle r_i \rangle^2\right)\left(\langle r_j^2 \rangle - \langle r_j \rangle^2\right)}}$$

- With: F(t), futures prices at t
- Correlation matrix C, (NxN), symmetric

From correlations to distances

- Non linear transformation
- Distances *d* between two nodes defined as follows:

$$d_{ij} = \sqrt{2\left(1-\rho_{ij}\right)}$$

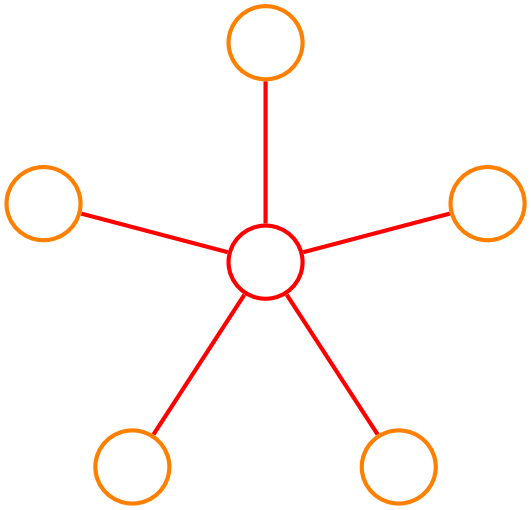
- Distance matrix D, (NxN)
- Full connected graph
  - represents all the possible connections between N nodes
  - weighted by the distances

Minimum spanning tree

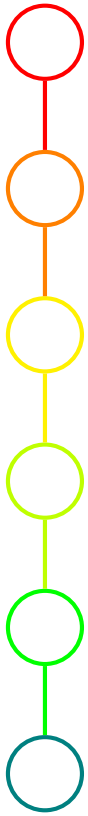
- All the nodes of the graph are spanned
- No loops
- Result: links of the MST are a subset of the initial graph
- The information space is reduced from (N(N-1)/2) to (N-1)
- In this study : shortest path linking all nodes

  Easiest path for the transmission of prices shocks

2. Topology of the MST



Star-like tree

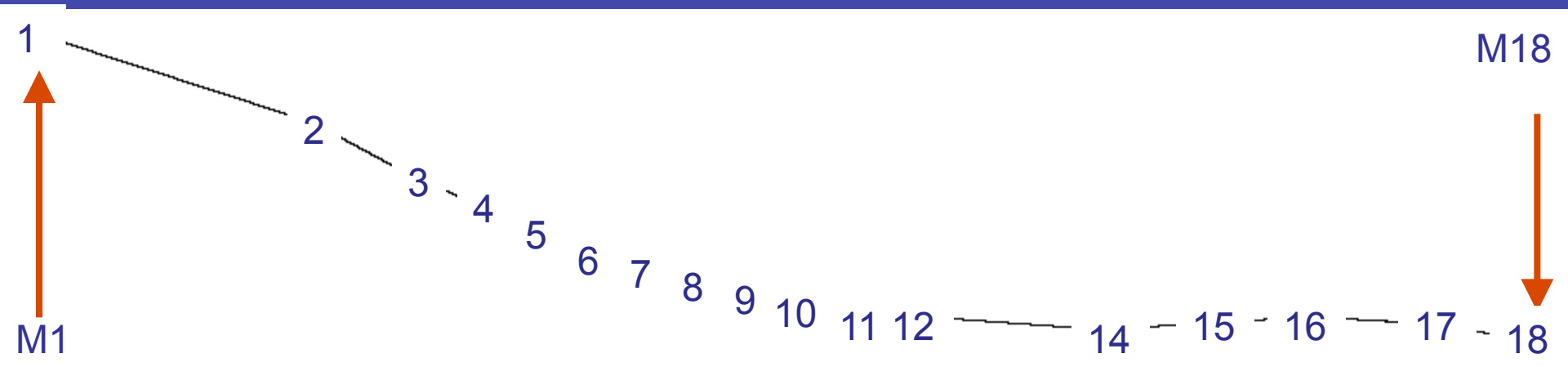


Chain-like tree

2. Topology of the MST

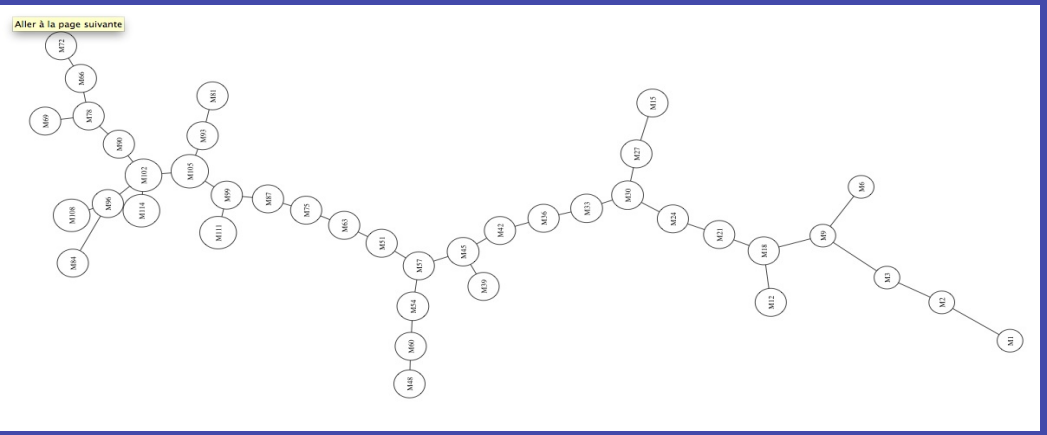
2.1. Maturity dimension

Heating oil – Month 1 to 18



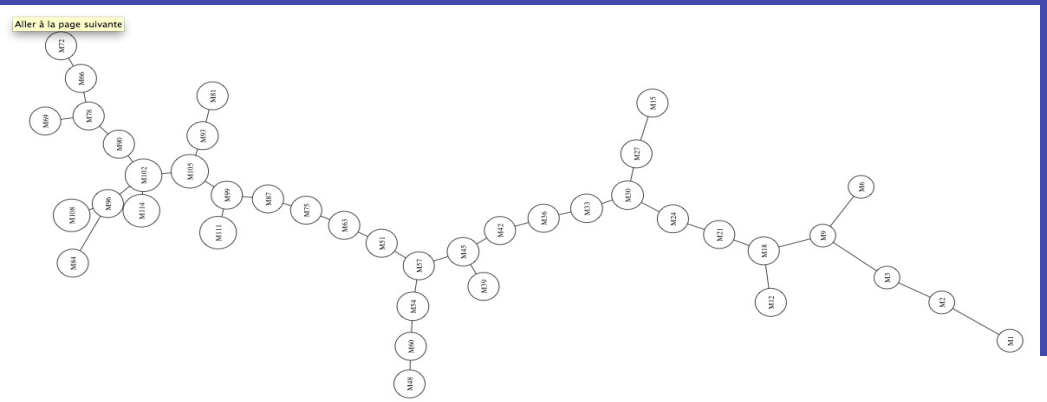
Samuelson effect

Evolution of the integration through time : Eurodollar



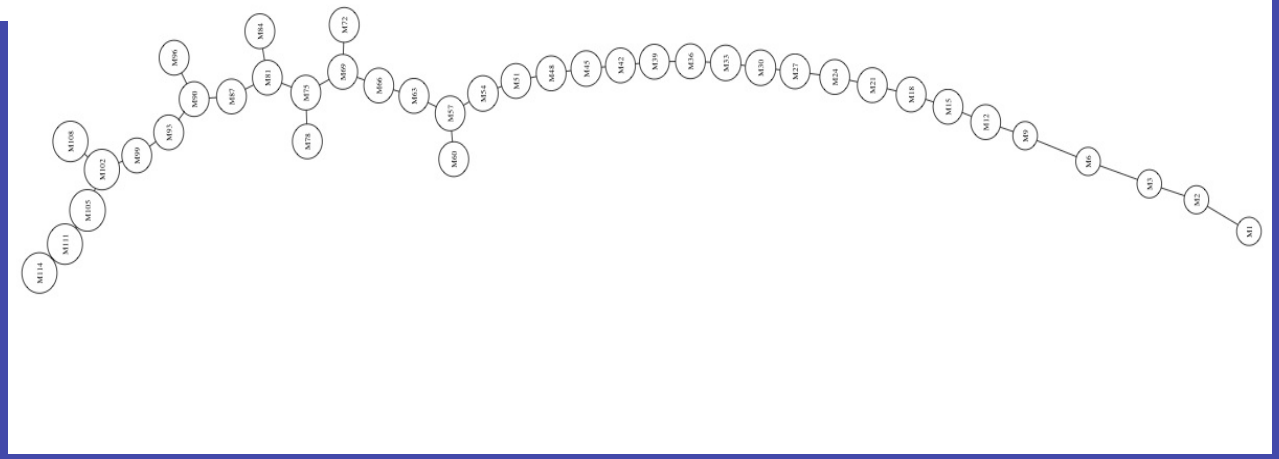
1998-2001

Evolution of the integration through time : Eurodollar



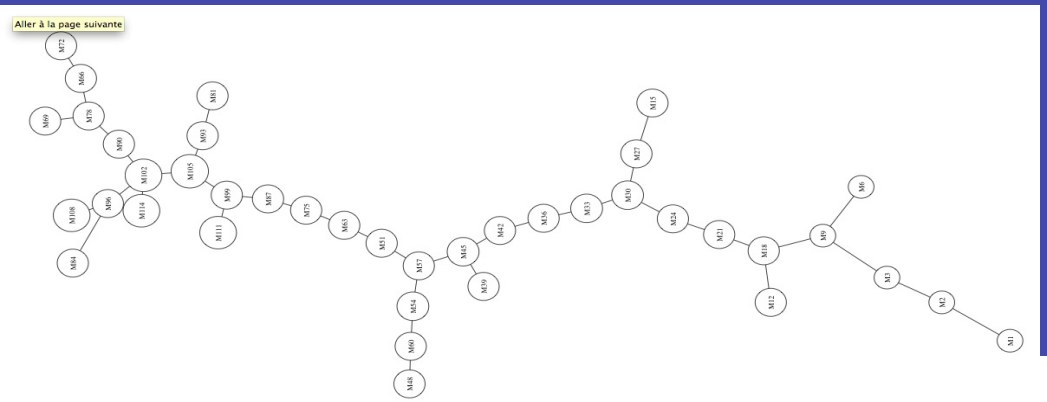
1998-2001

2001-2004



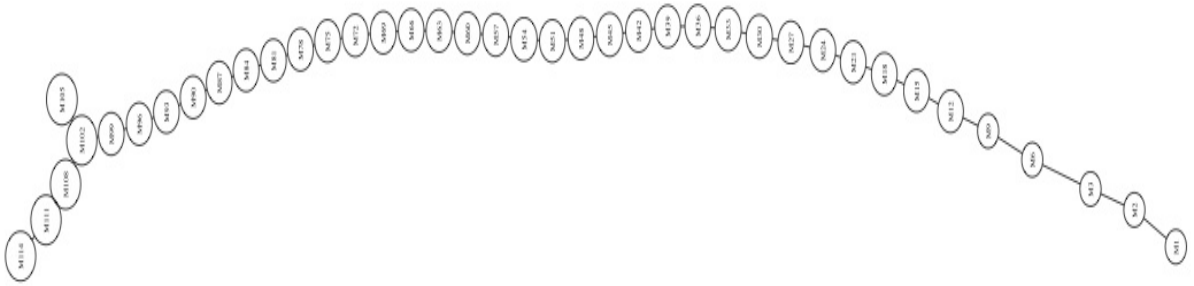
Evolution of the integration through time : Eurodollar

1998-2001

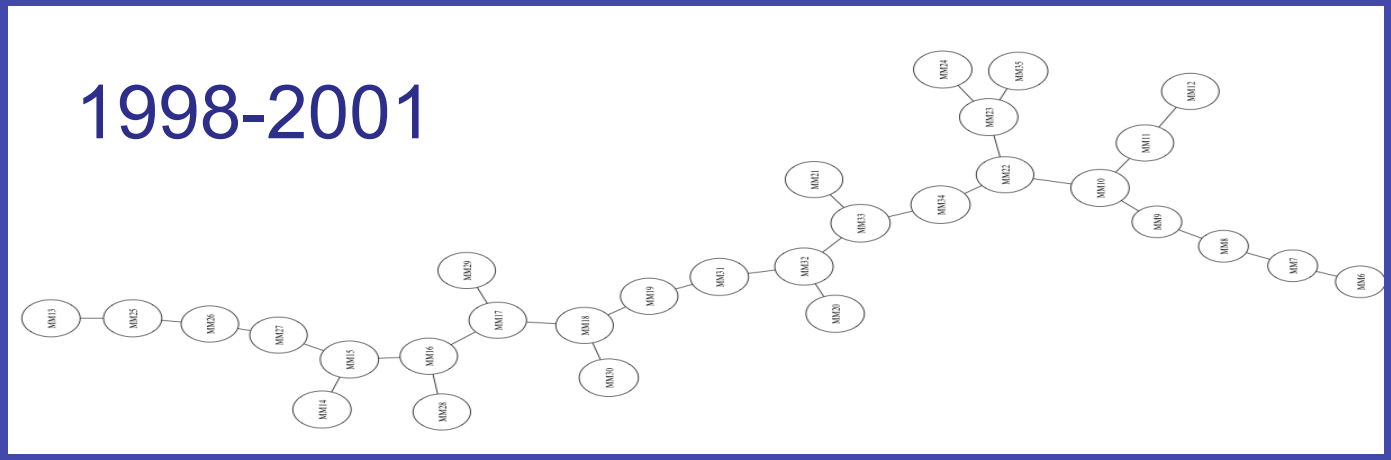


2001-2004

2004-2009



Evolution of the integration through time, US natural gas



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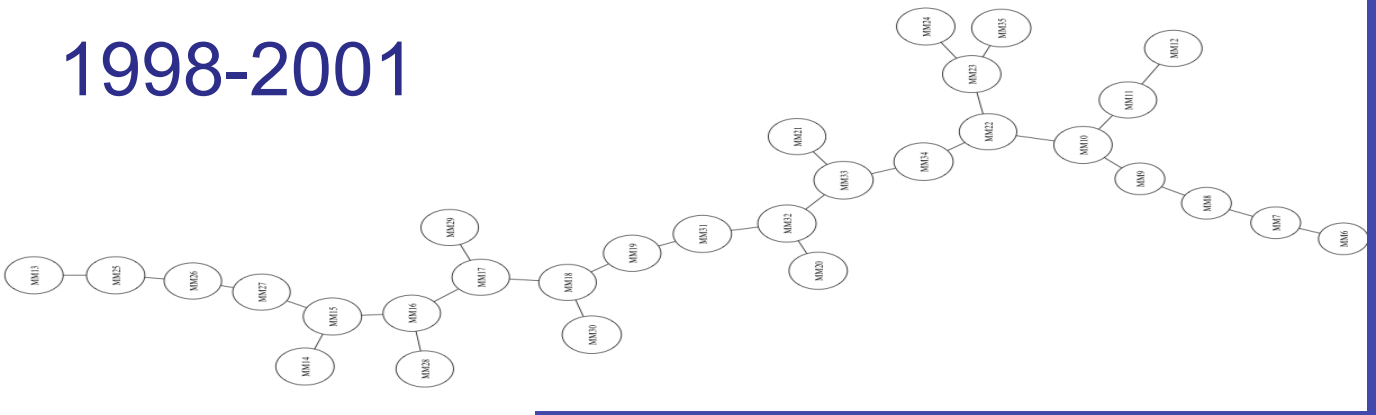
1998-2001



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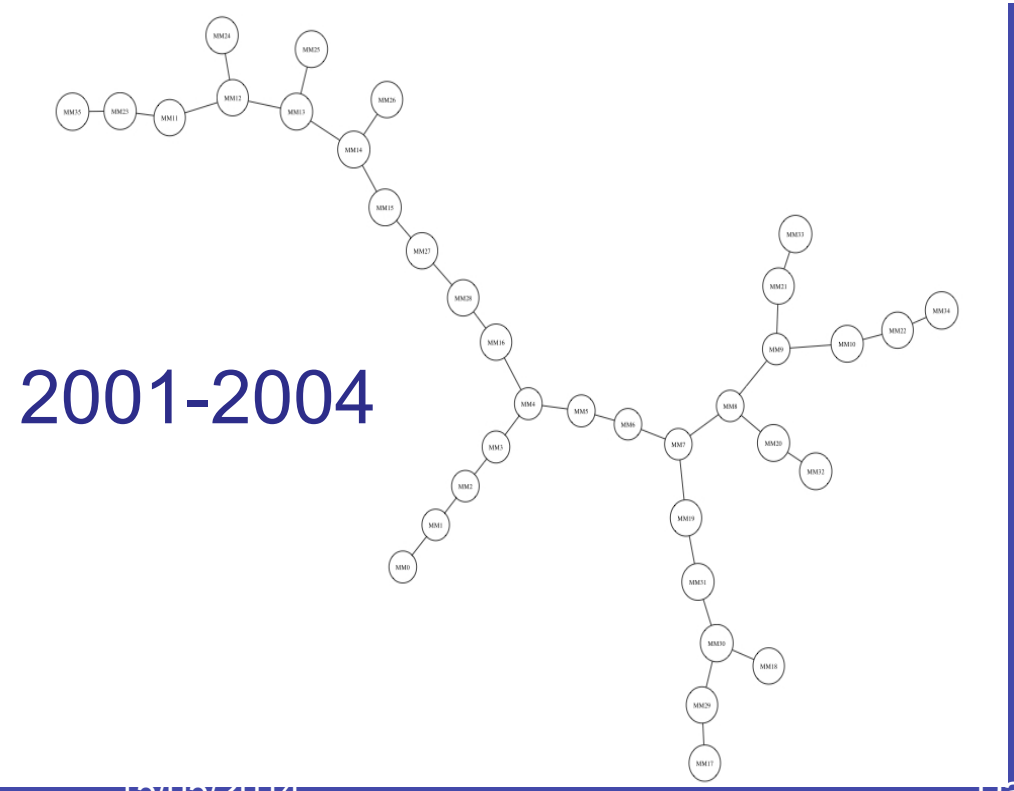

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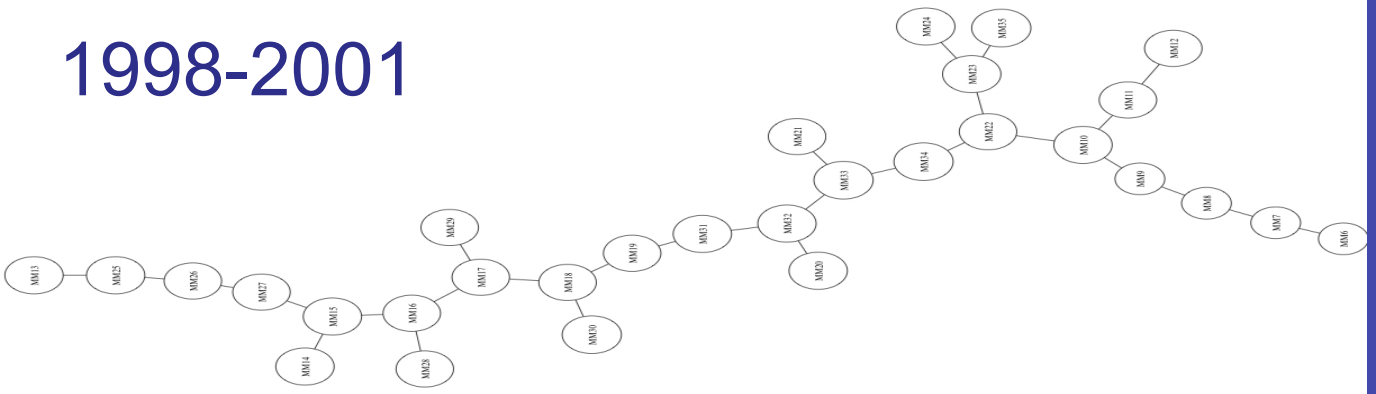

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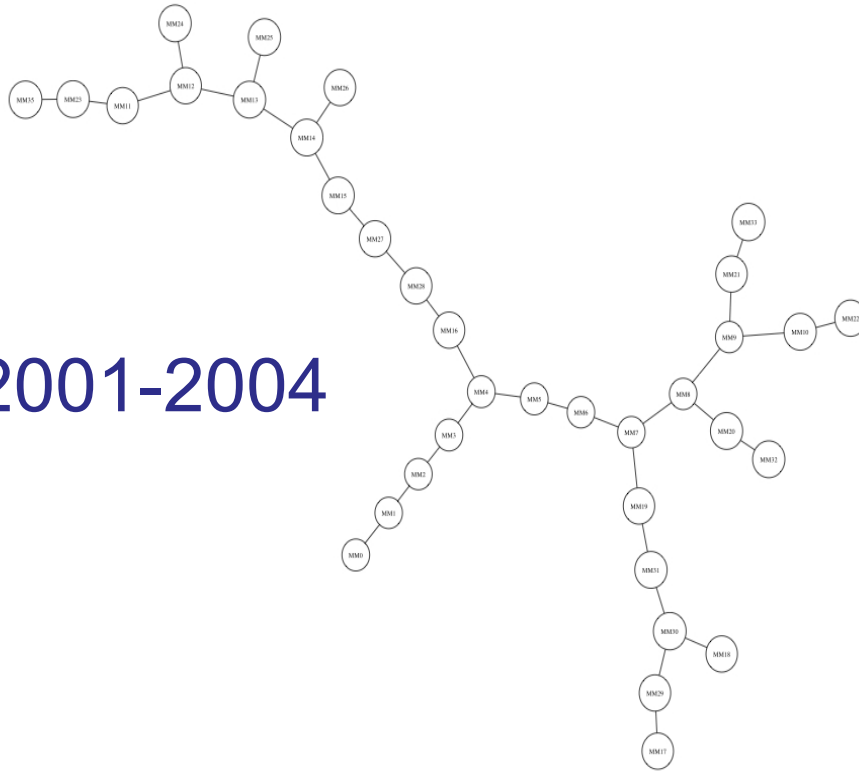
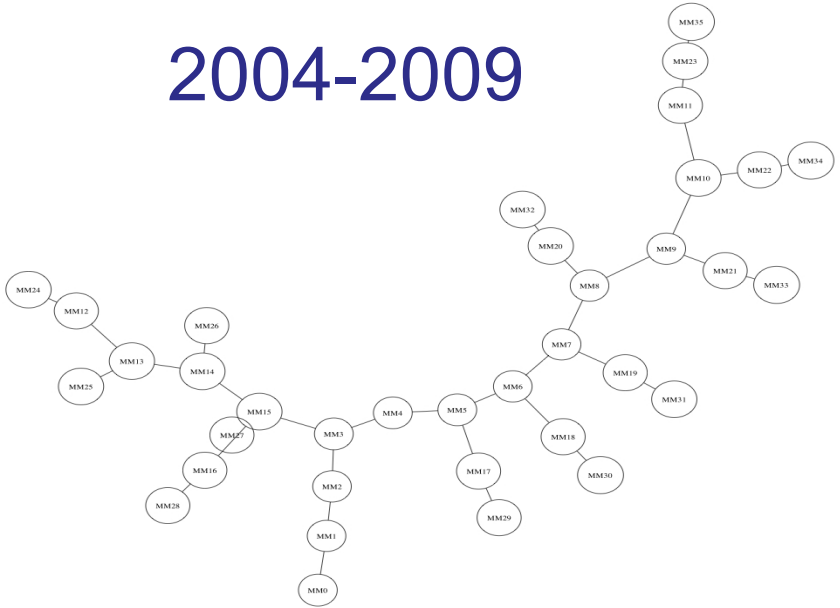
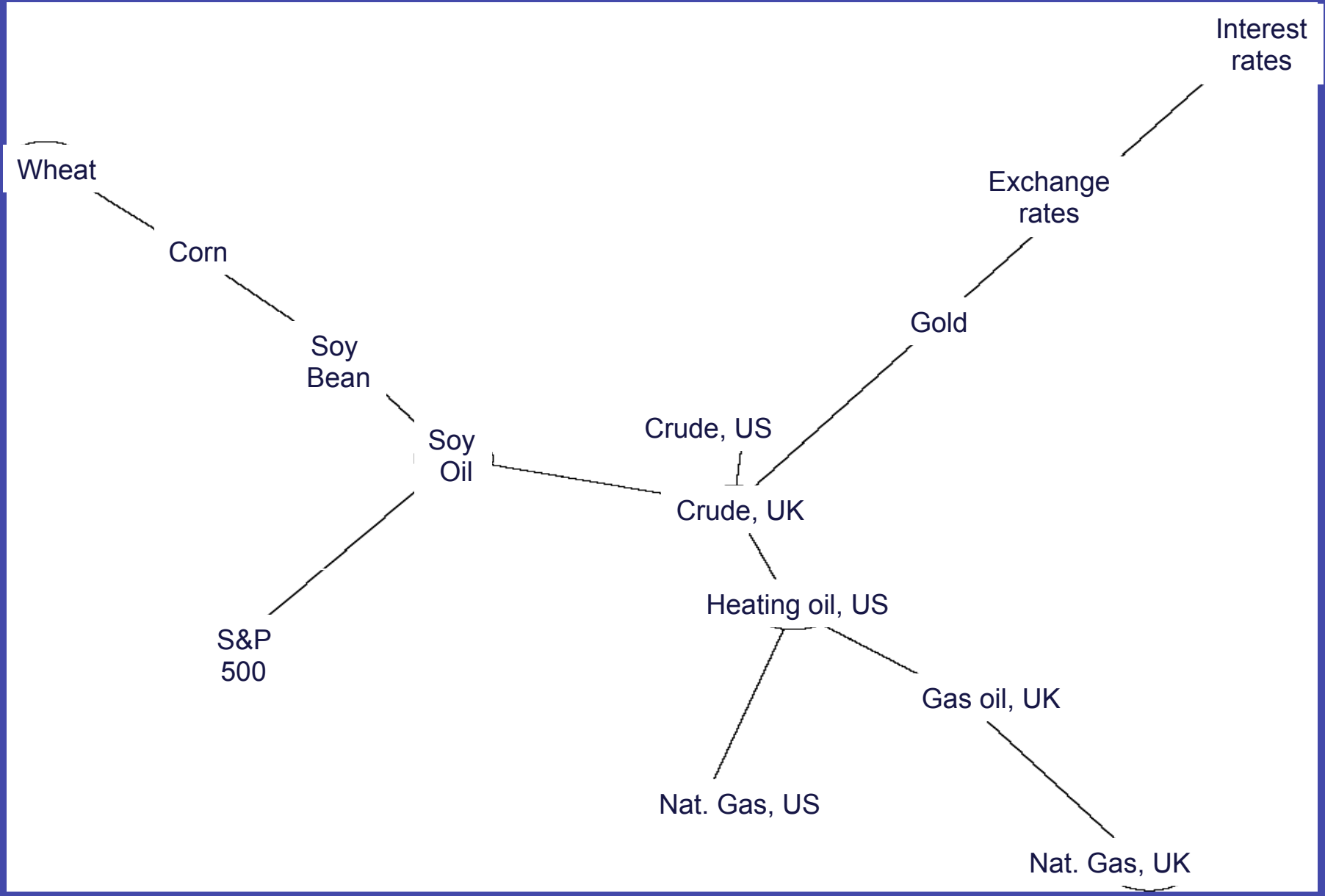


2004-2009



2.2. Spatial dimension



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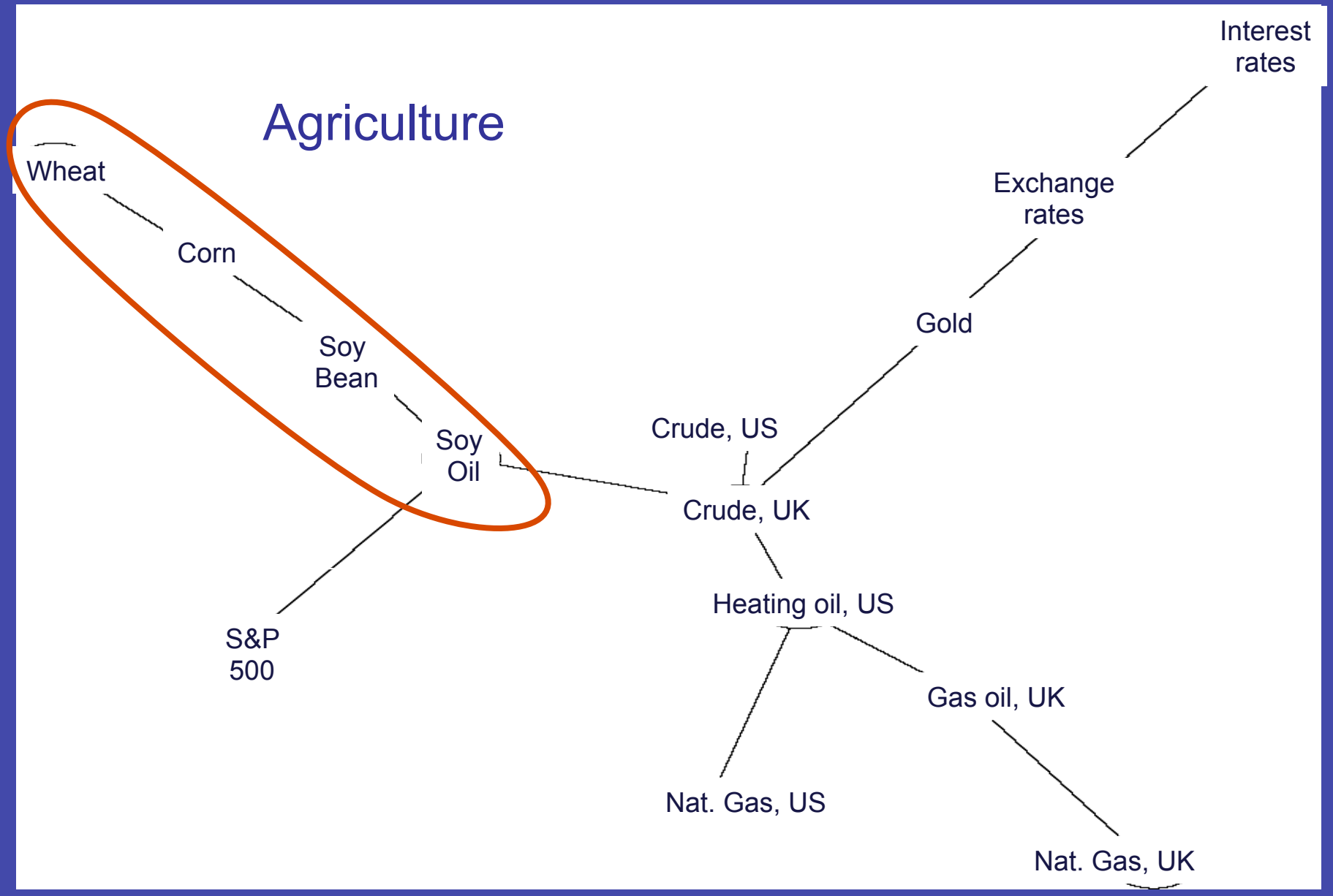

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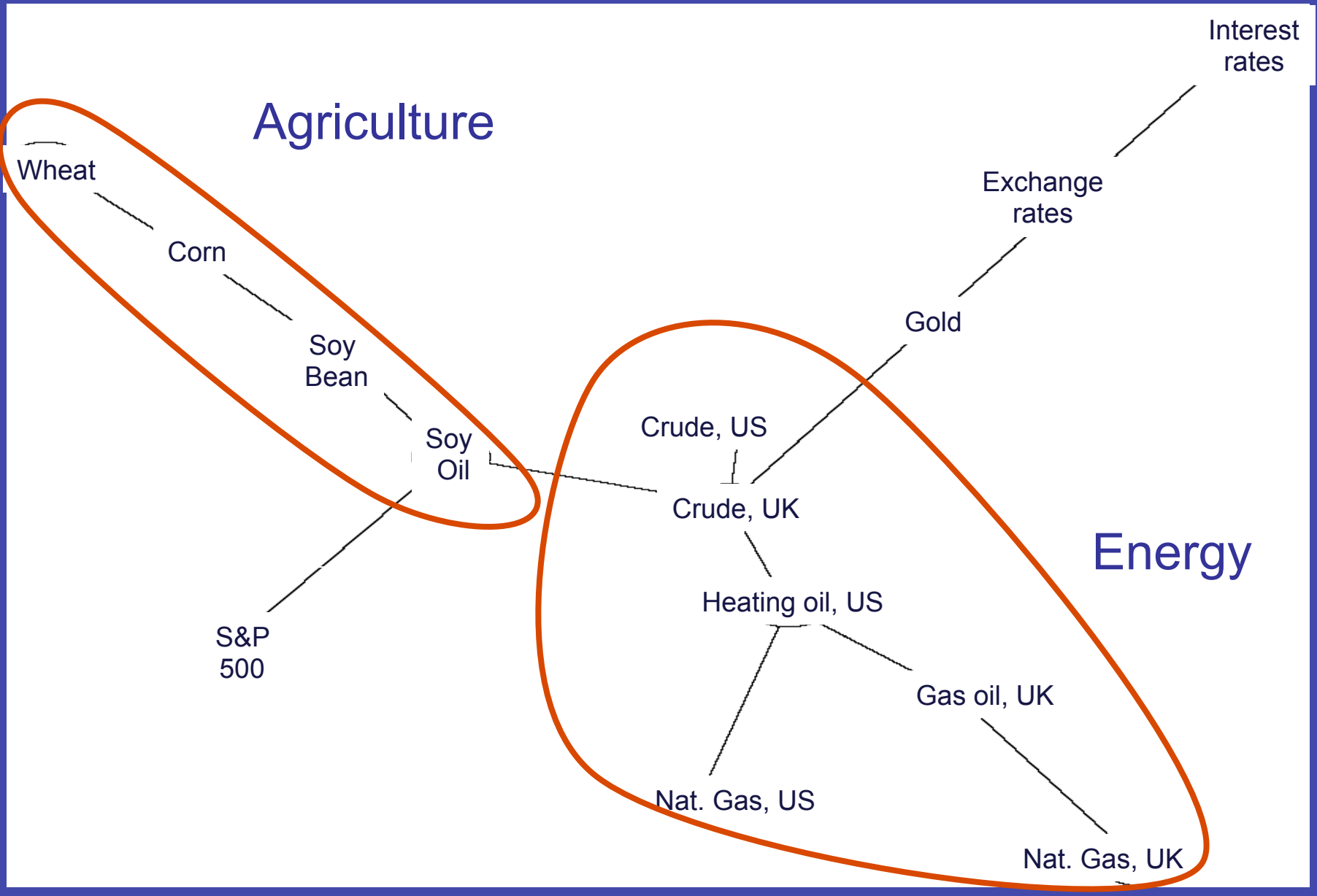

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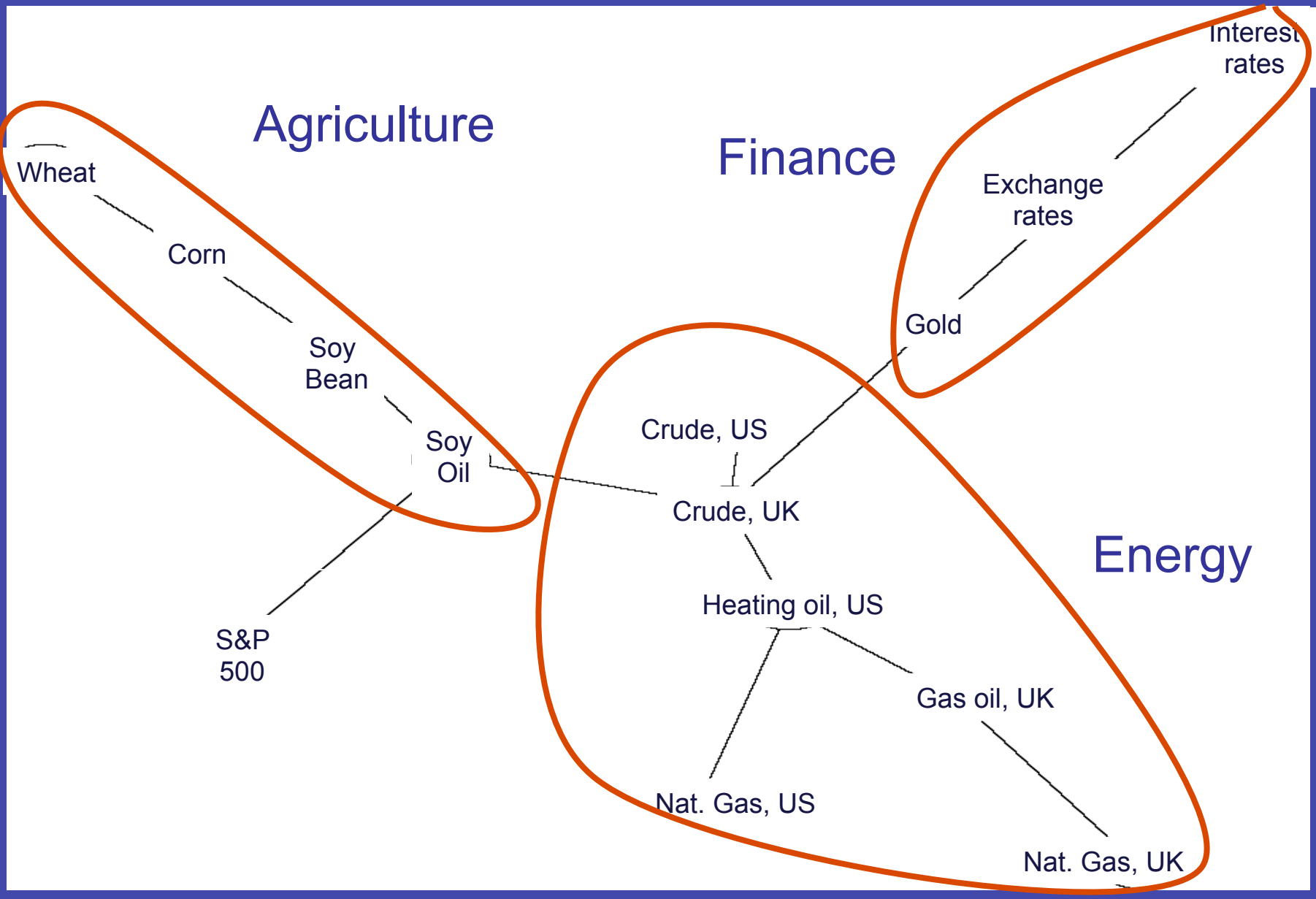

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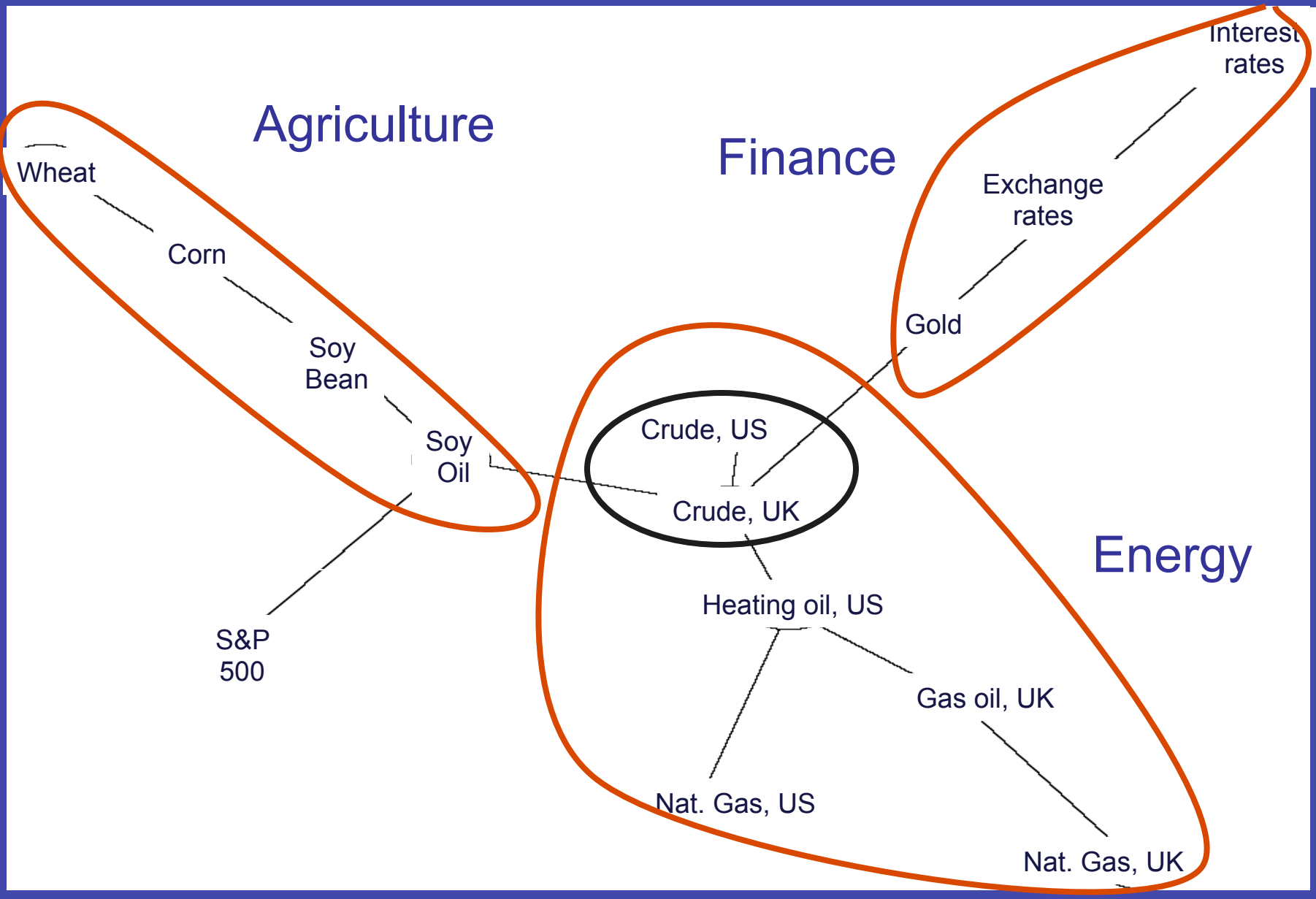

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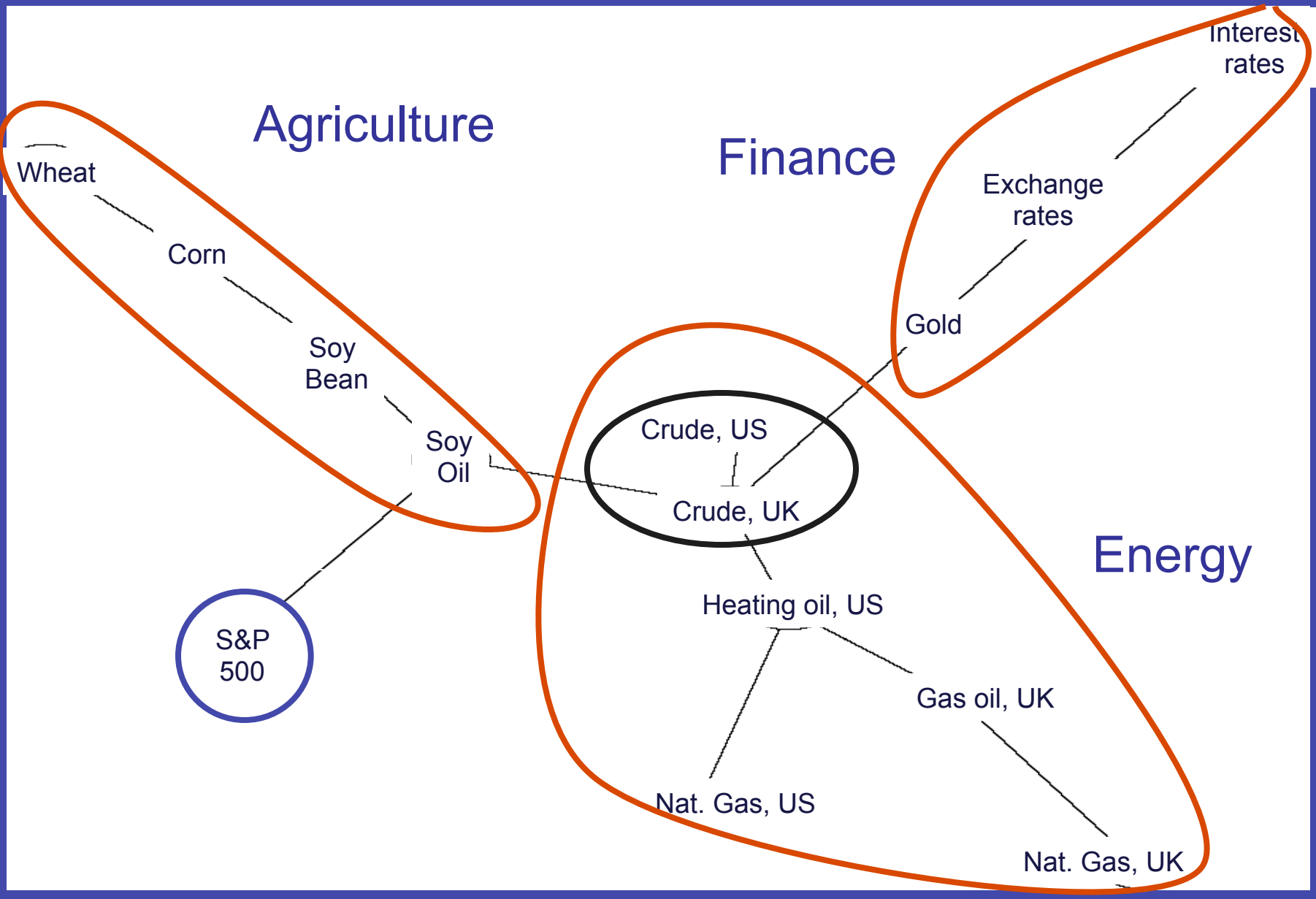

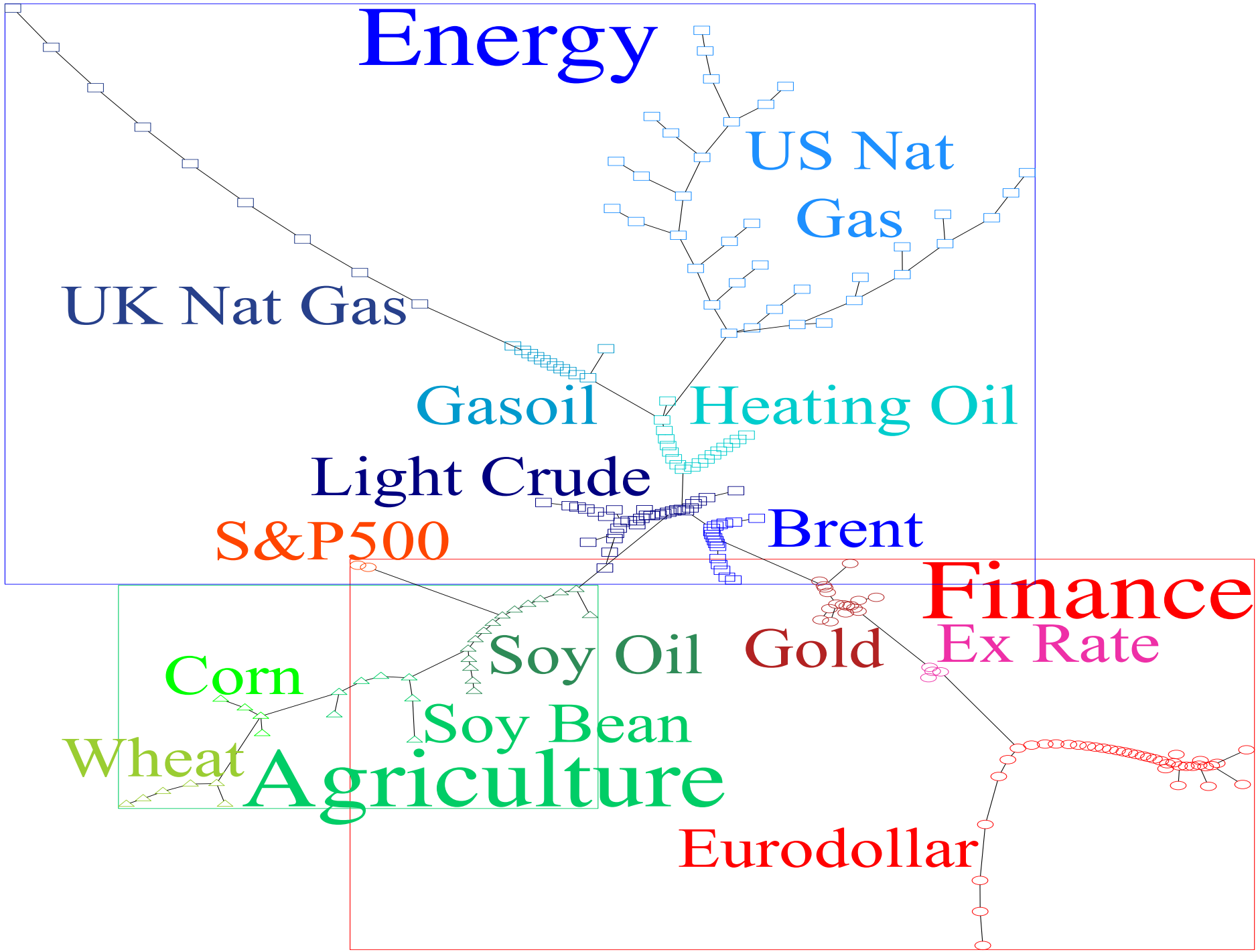
Energy
US Nat Gas
UK Nat Gas
Gasoil
Heating Oil
Light Crude
Brent
S&P500
Finance
Gold
Ex Rate
Soy Oil
Corn
Soy Bean
Wheat
Agriculture
Eurodollar

2.4 Allometric coefficients

- Quantifying the degree of linearity in the trees
- The root is the node with the highest connectivity
- Starting from this root, two coefficients A_i and B_i are assigned to each node i:

$$A_i = \sum_j A_j + 1 \qquad B_i = \sum_j B_j + A_i$$

$$B \sim A^{\eta}$$

Where η is the allometric exponent
η stands between 1+ (star-like) and 2- (chain-like)

MATURITIES	Static	CI95%	Dynamic	CI95%
Light crude	1.994	1.9058 – 2.0822	1.910	1.8904 – 1.929
Brent crude	1.889	1.883 – 1.894	1.888	1.88 – 1.895
Heating oil	1.899	1.891 – 1.906	1.886	1.874 – 1.898
Gasoil	1.880	1.874 – 1.885	1.845	1.835 – 1.854
Nat. gas (US)	1.750	1.677 – 1.822	1.796	1.745 – 1.847
Nat. Gas (Eu)	1.874	1.87 – 1.877	1.832	1.83 – 1.834
Wheat	1.864	1.609 – 2.118	1.761	1.694 – 1.827
Soy bean	1.848	1.661 – 2.034	1.680	1.623 – 1.736
Soy oil	1.889	1.883 – 1.894	1.856	1.832 – 1.879
Corn	1.880	1.874 – 1.885	1.772	1.731 – 1.813
Eurodollar	1.927	1.817 – 2.036	1.846	1.806 – 1.885
Gold	1.732	1.552 – 1.912	1.826	1.788 – 1.863
SPATIAL	1.493	1.383 – 1.602	1.621	1.574 – 1.668
3D	1.757	1.712 – 1.802	1.850	1.673 – 2.023

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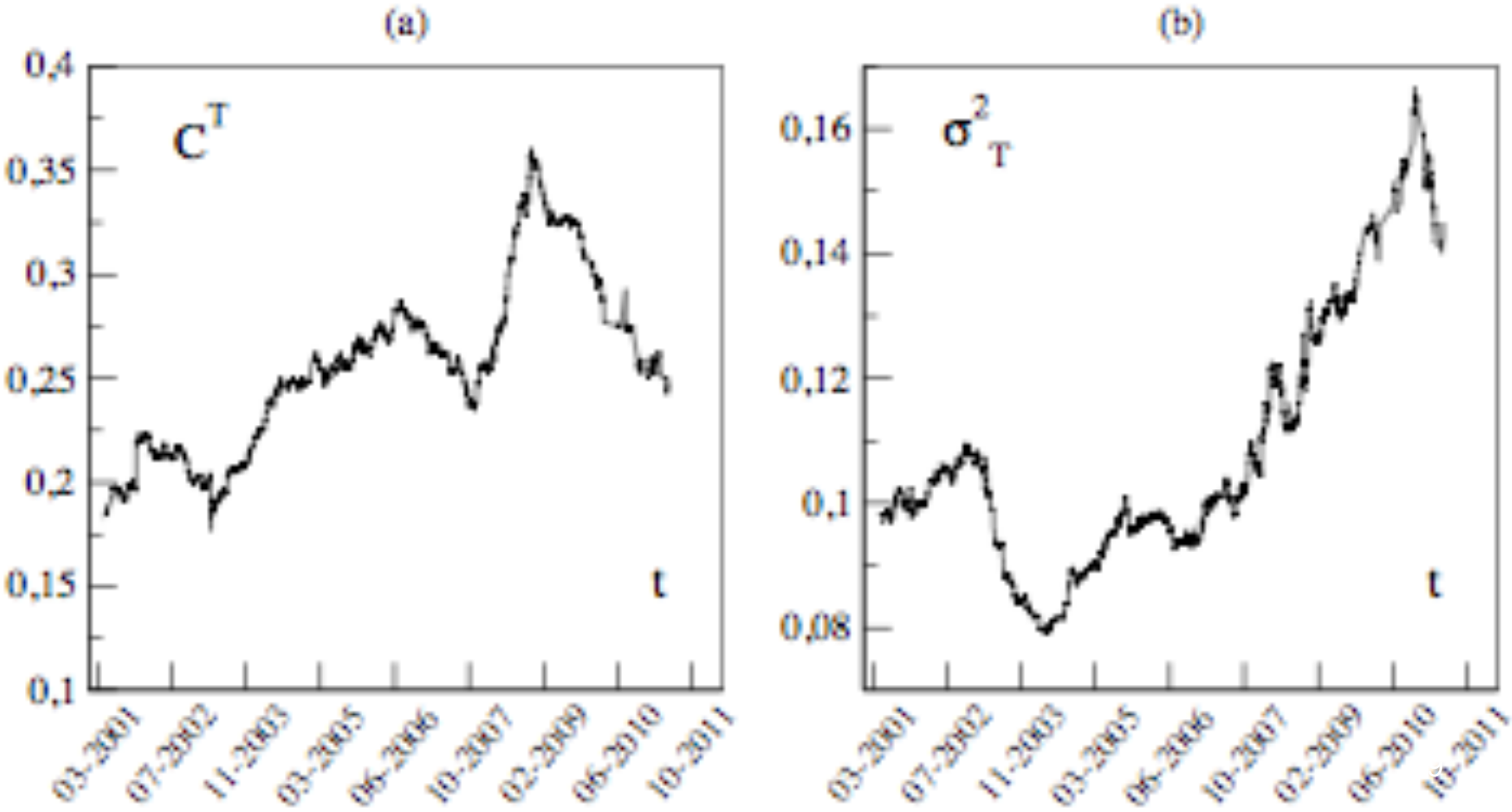
3. Dynamical studies

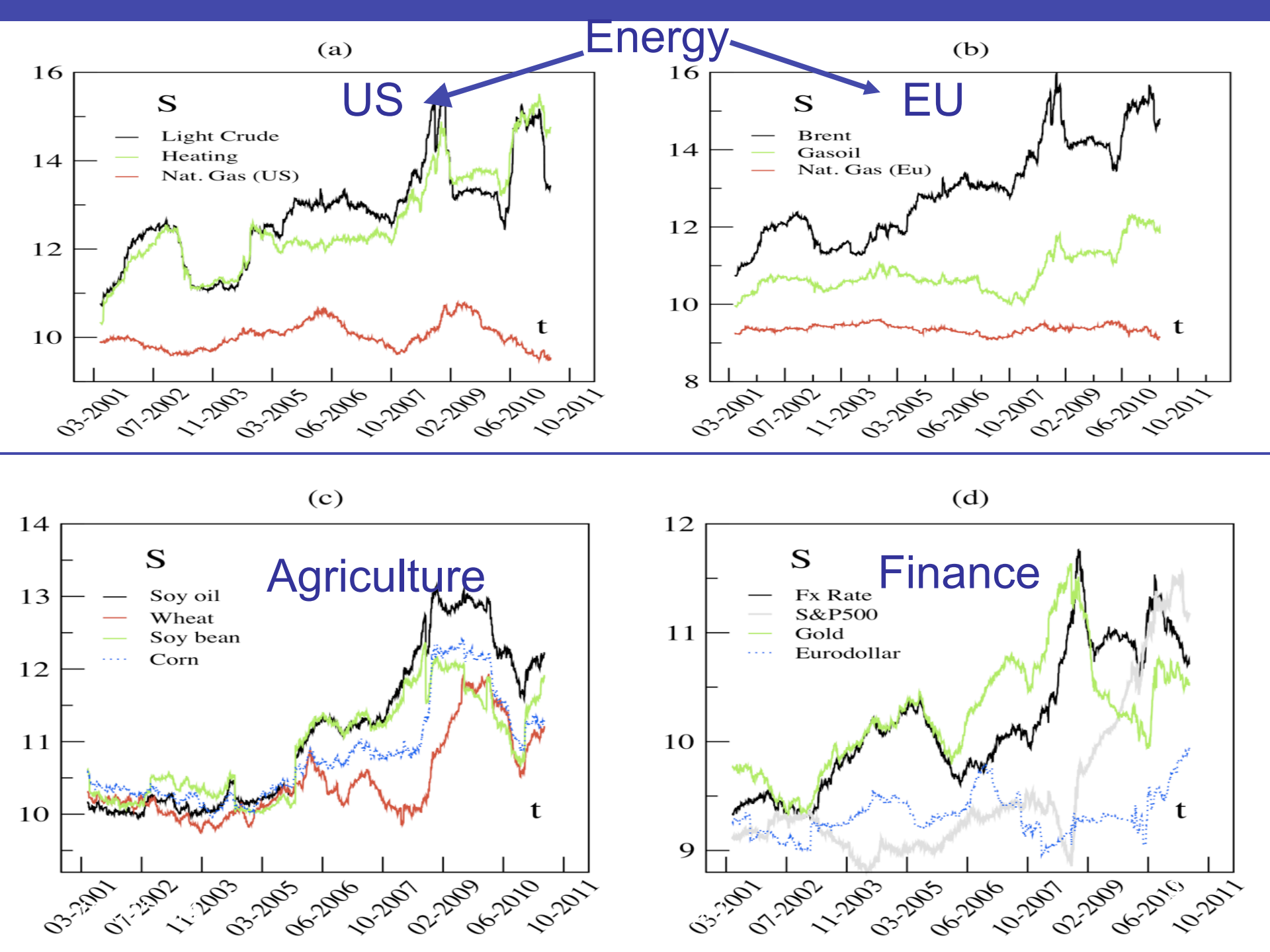
3.1. Full connected graph : mean correlations and their variances (3-D)



3.2. Node's strength

- Full connected graph
- The node's strength S_i indicates the closeness of one node i to all others:

$$S_i = \sum_{i \neq j} \frac{1}{d_{ij}}$$


Energy
(a)
US
S
Light Crude
Heating
Nat. Gas (US)
t
16
14
12
10
03-2001
07-2002
11-2003
03-2005
06-2006
10-2007
02-2009
06-2010
10-2011
(b)
EU
S
Brent
Gasoil
Nat. Gas (Eu)
t
16
14
12
10
8
03-2001
07-2002
11-2003
03-2005
06-2006
10-2007
02-2009
06-2010
10-2011
(c)
Agriculture
S
Soy oil
Wheat
Soy bean
Corn
t
14
13
12
11
10
03-2001
07-2002
11-2003
03-2005
06-2006
10-2007
02-2009
06-2010
10-2011
(d)
Finance
S
Fx Rate
S&P500
Gold
Eurodollar
t
12
11
10
9
03-2001
07-2002
11-2003
03-2005
06-2006
10-2007
02-2009
06-2010
10-2011

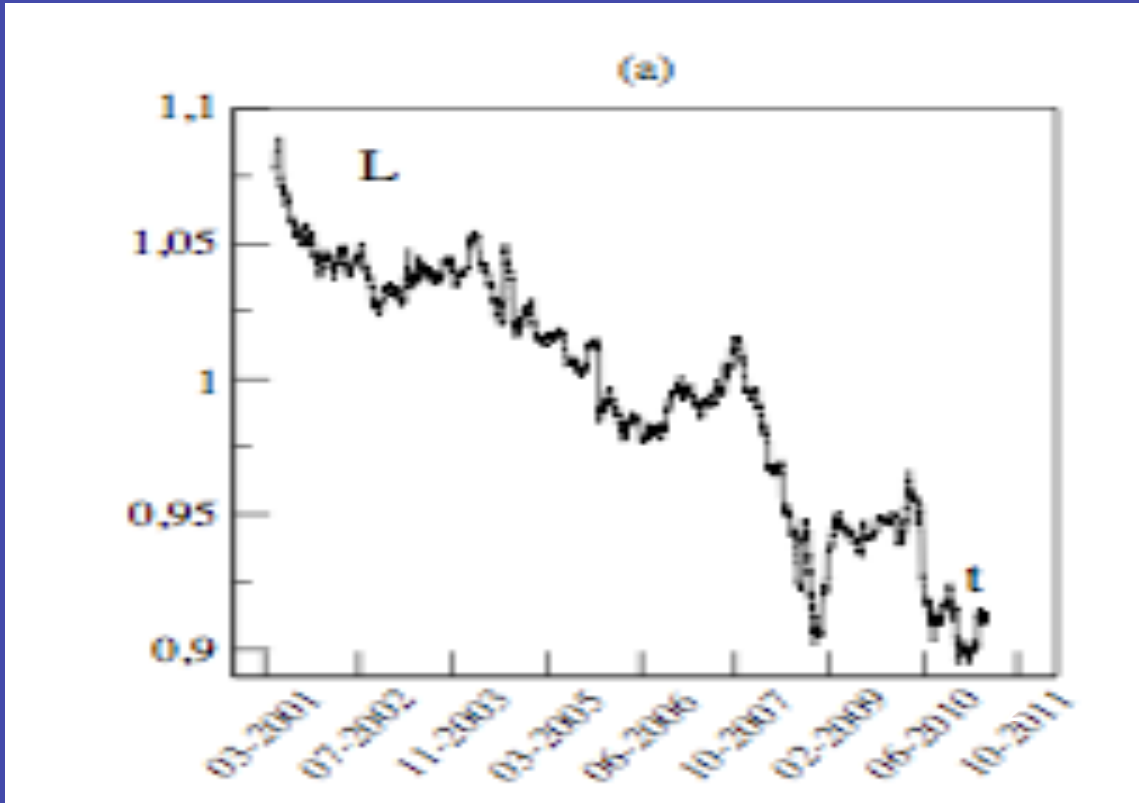
3.3. Normalized tree's length

- Sum of the lengths of the links belonging to the MST:

$$L(t) = \frac{1}{N-1} \sum_{(i,j) \in MST} d_{ij}$$

  - The more the length shortens, the more integrated the system is

Spatial dimension



3.4. Survival ratios

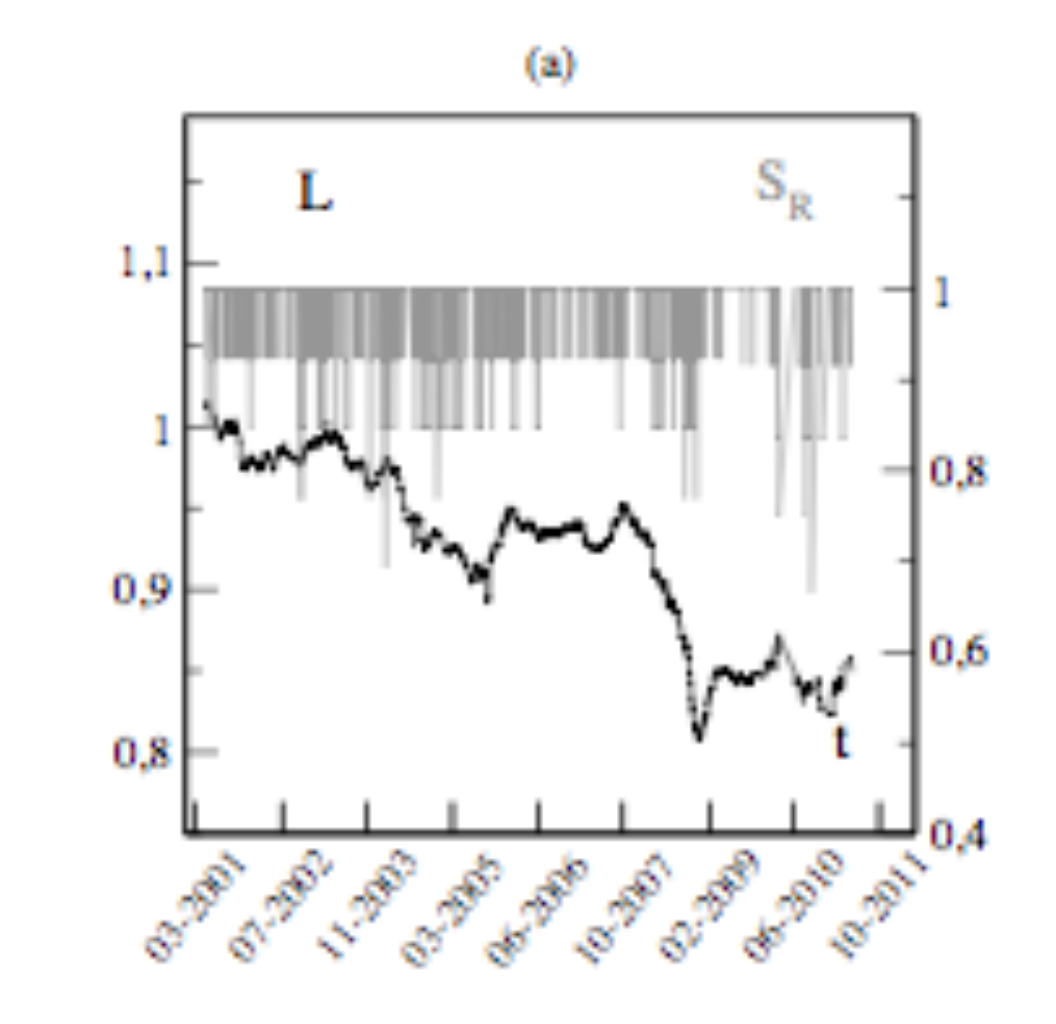
- Robustness of the topology over time
- The survival ratio S_R refers to the fractions of edges in the MST, that survives between two consecutive trading days:

$$S_R(t) = \frac{1}{N-1} |E(t) \cap E(t-1)|$$

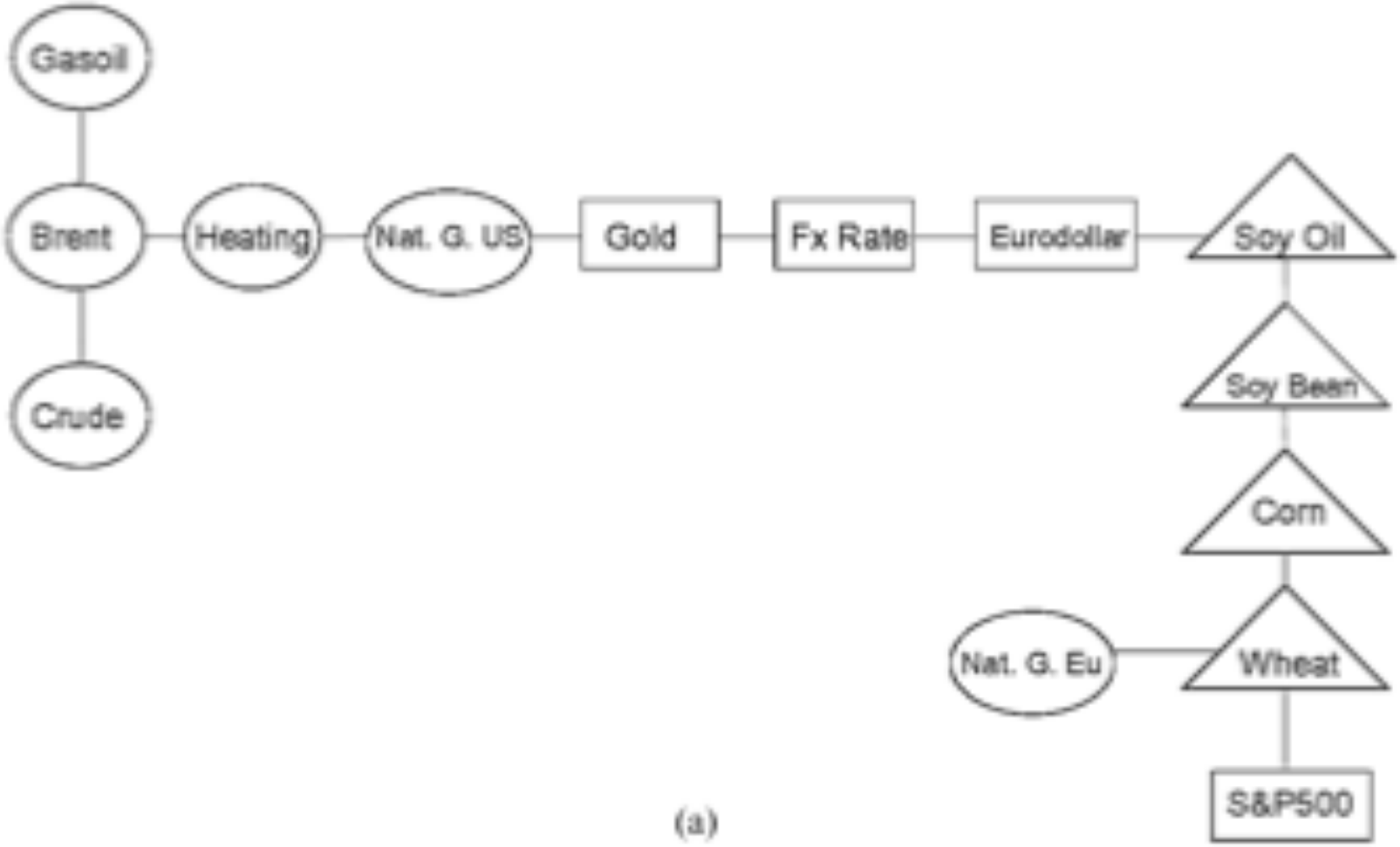
E(t) : set of edges at t

Pruning the trees

- Analysis of inter-market and inter-sectors reorganizations
- Consider only the links between markets, whatever the maturity is considered

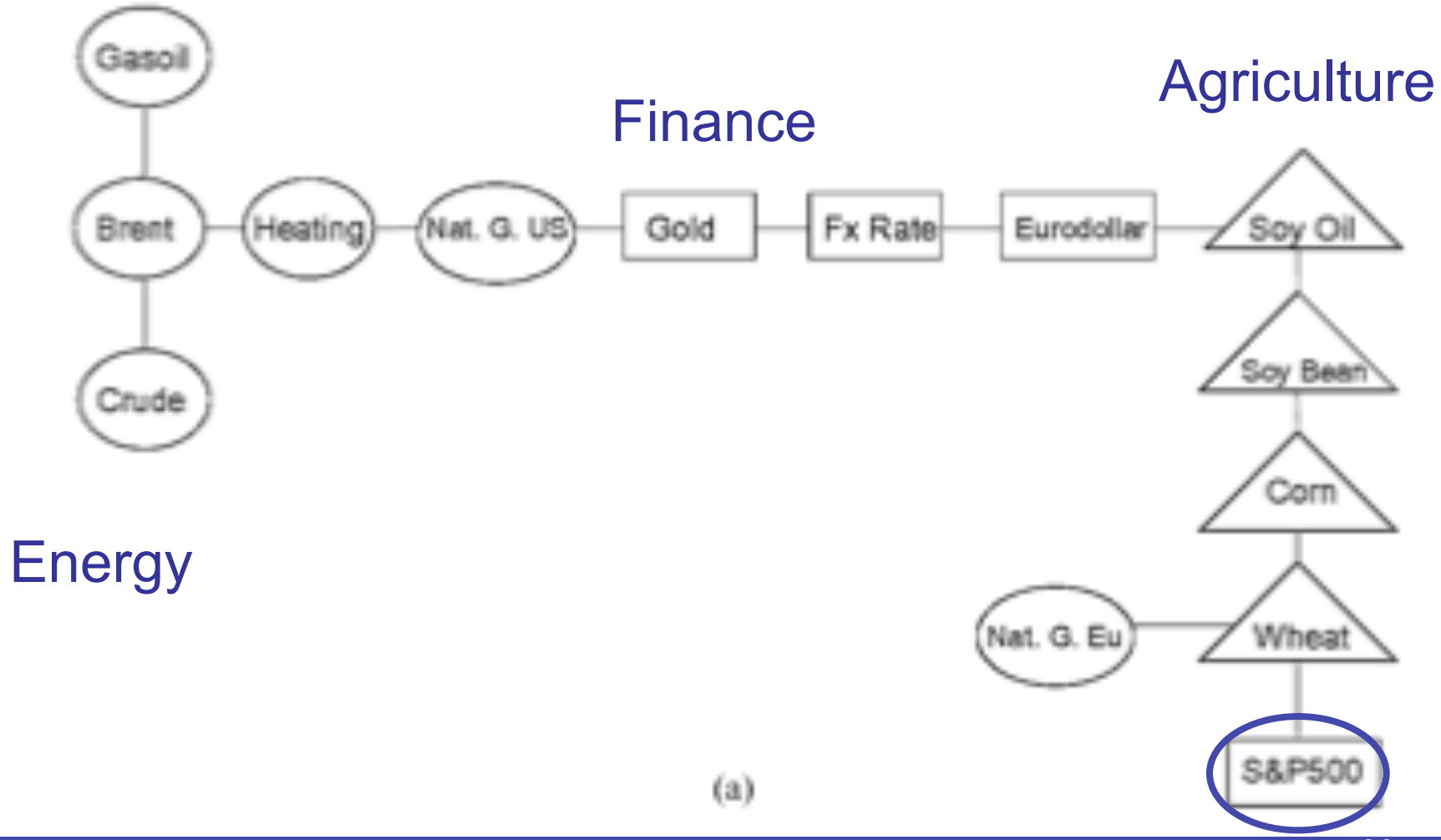


Pathological configuration : an example

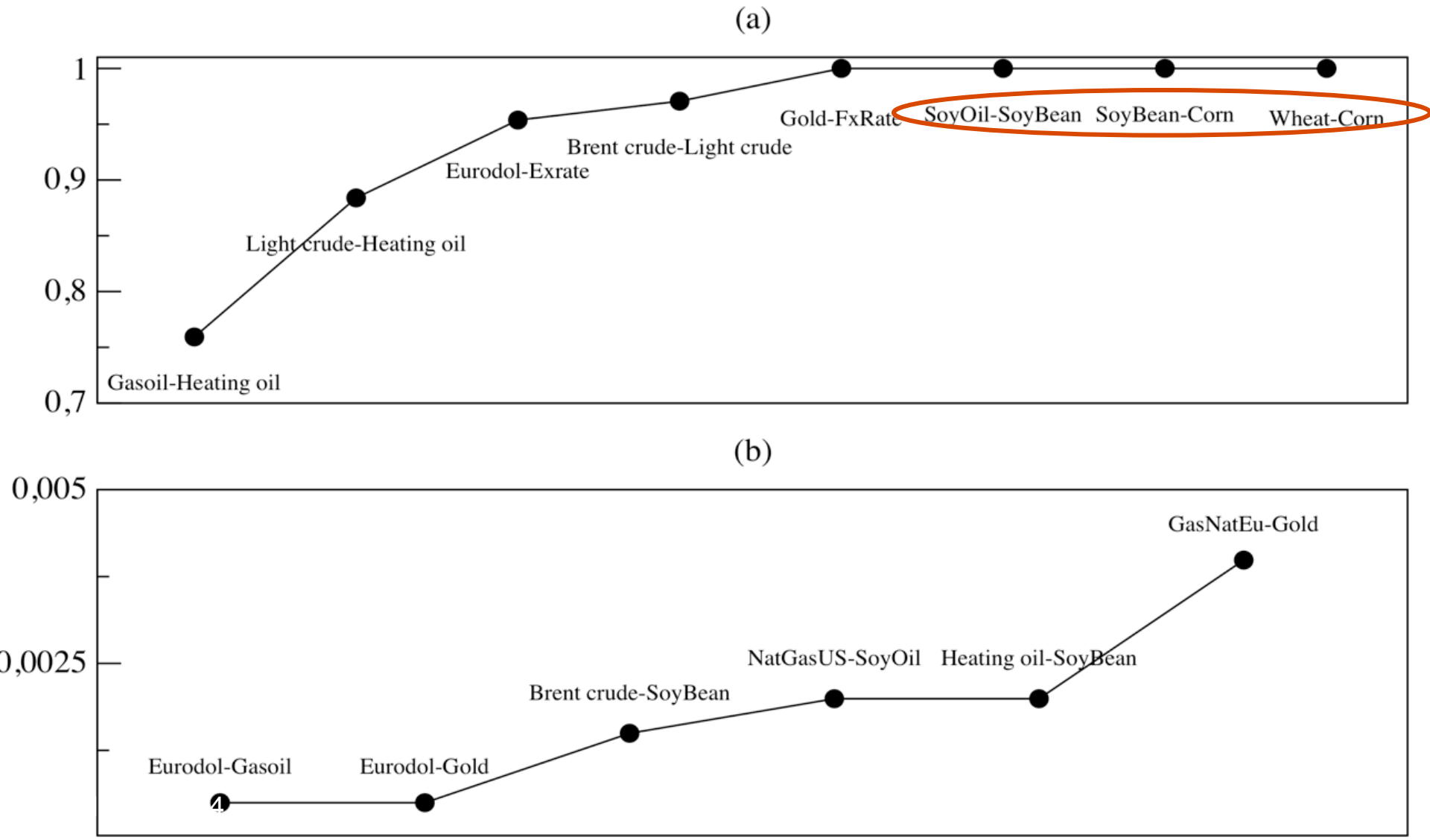


(a)

Pathological configuration : an example



Most stable links



Main results - Extensions

MAIN RESULTS

1. Topology

- Chain-like trees in the maturity dimension
- Star-like trees in the spatial and 3-D dimensions

2. Emerging taxonomy

- Trees organized around the three sectors of activity
- Center of the graph : two crude oils

3. Integration

- Increases in all dimensions (spatial, maturity, 3D)
- Progresses at the heart of the system

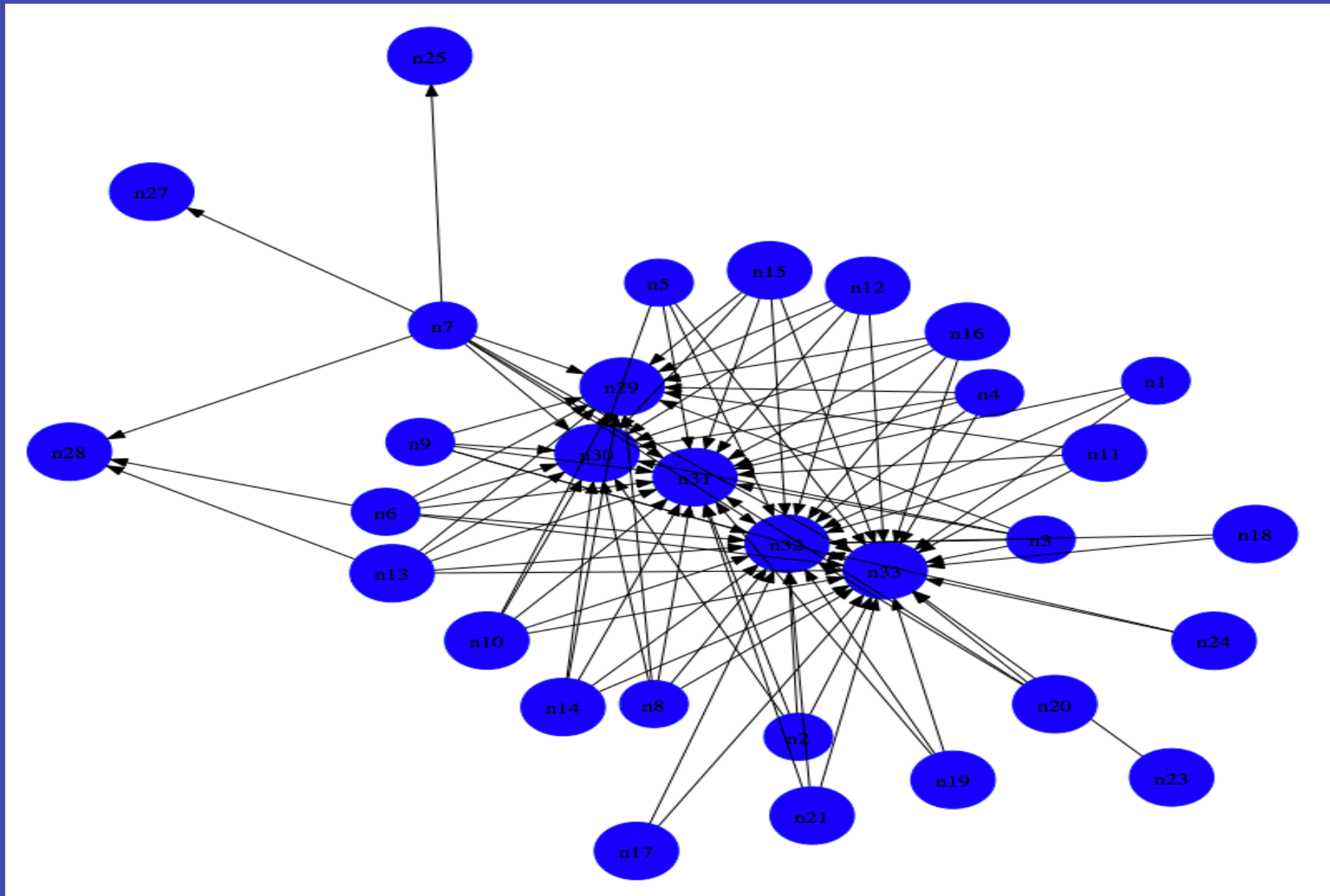
EXTENSIONS

1. Introducing directions in the graph
2. Event studies / financial crises

1. Introducing directions in the graph

- Full connected graph
- Information flows :
  - static analysis
  - dynamic analysis

Full connected graph, maturity dimension



Information flows: static analysis

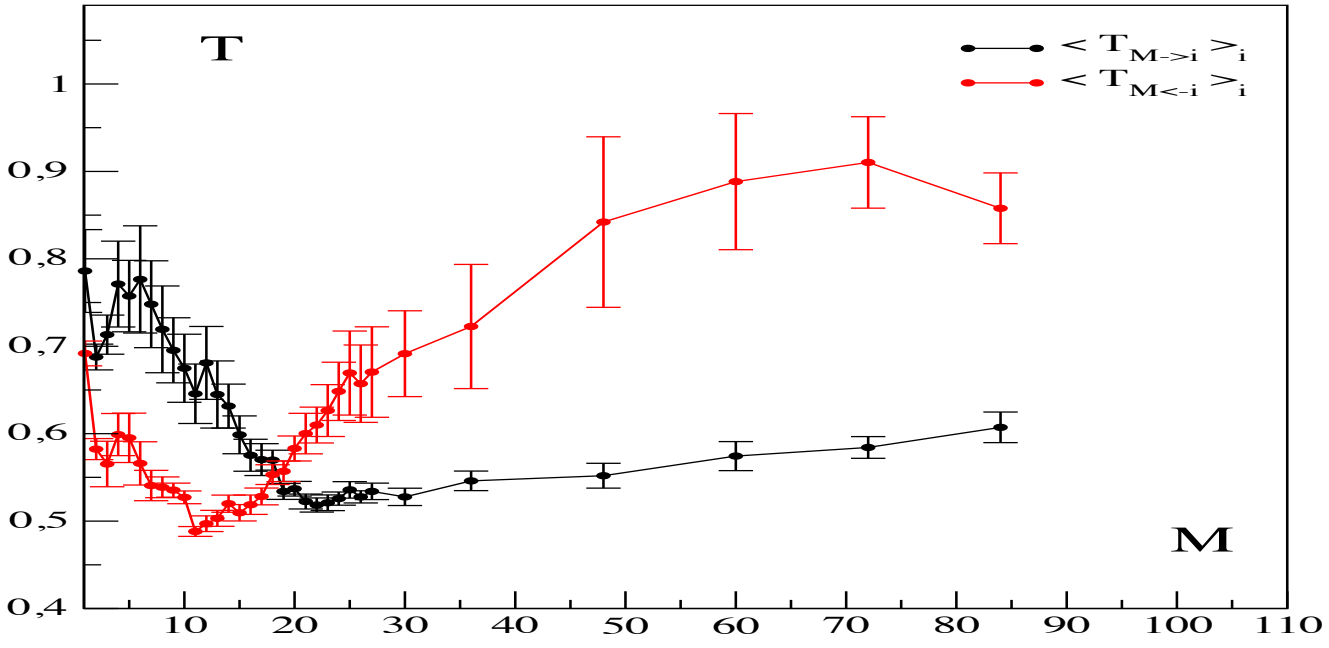


Figure: Average information transfer between maturities, 2001-2011

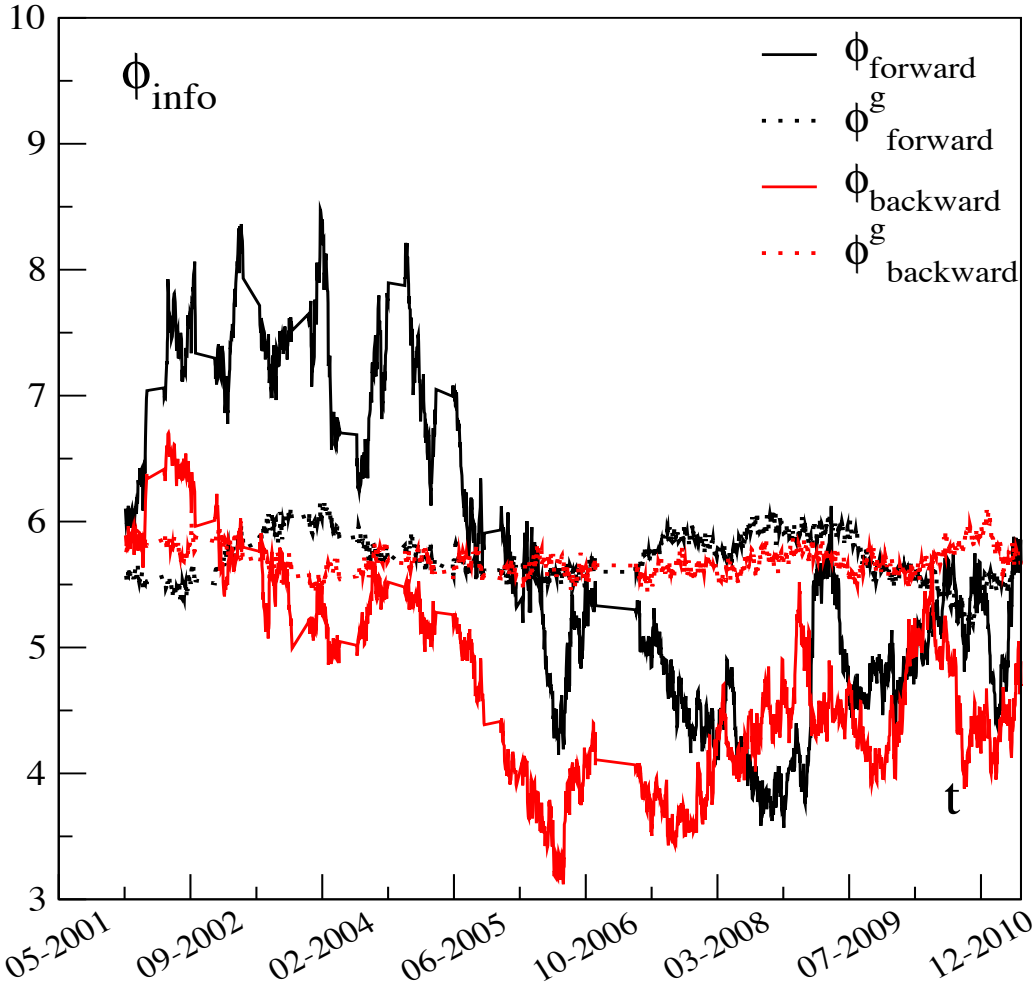
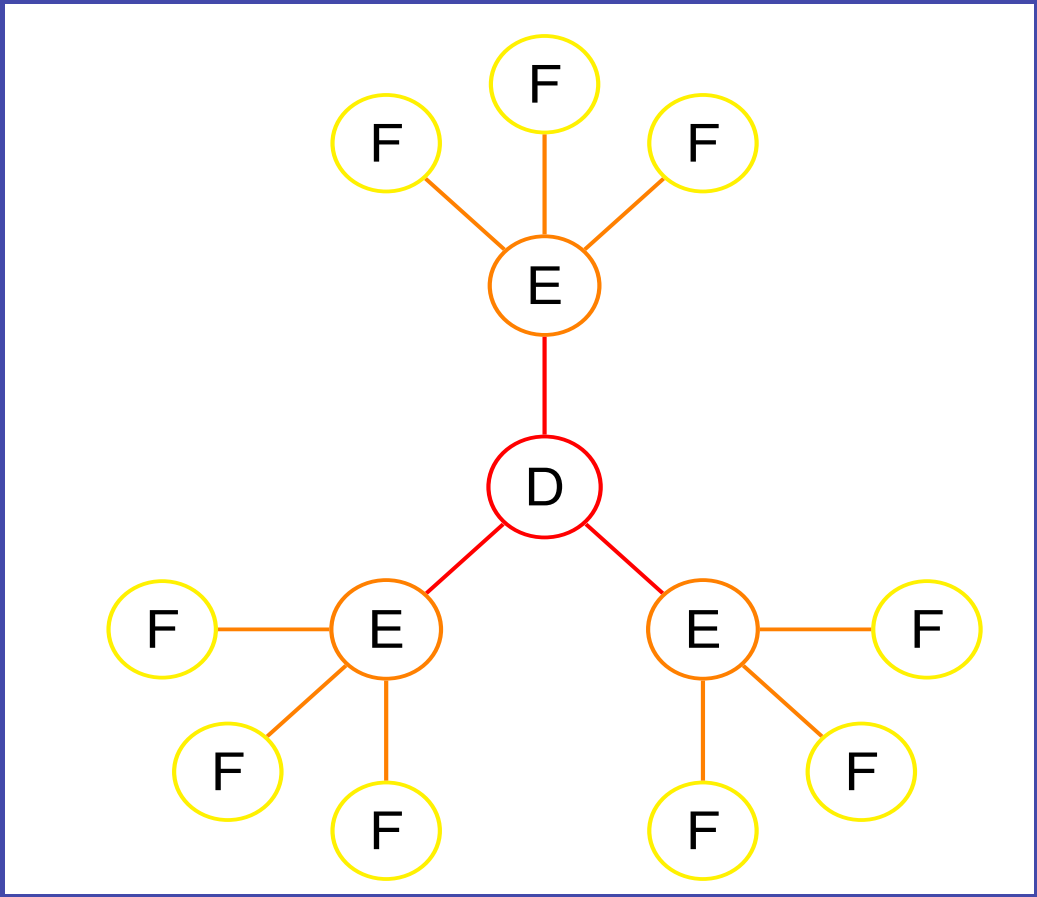
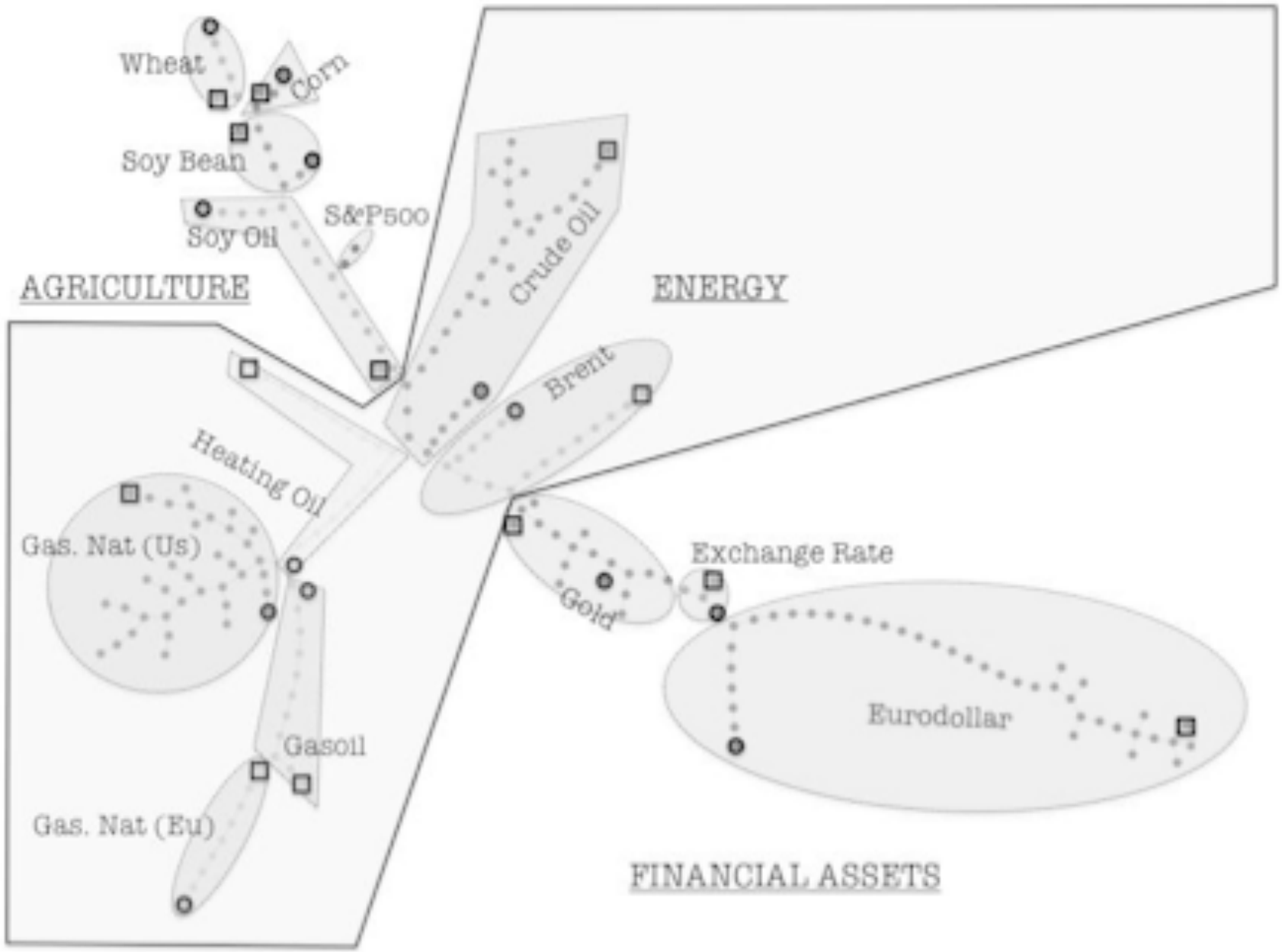


Figure: Information transfer between maturities, 2001-2011

2. Event studies / financial crises

Centrality measures




Wheat
Corn
Soy Bean
Soy Oil
S&P500
Crude Oil
AGRICULTURE
ENERGY
Brent
Heating Oil
Gas. Nat (Us)
Exchange Rate
Gold
Eurodollar
Gasoil
Gas. Nat (Eu)
FINANCIAL ASSETS