How Fukushima-Daiichi core meltdown changed the probability of nuclear accidents?

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Motivation

1. Two weeks after Fukushima Daiichi an article in a French newspaper claimed that the probability of a major nuclear accident in the next 30 years is greater than 50% in France and more than 100% in Europe!
   - What is the probability of a new accident tomorrow?
2. Probabilistic Risk Assessment (PRA) carried out by nuclear vendors and operators estimate a frequency of core melt down (CDF) about 2.0E-5 per reactor year
3. This means one accident per 50.000 reactor years, however the observed frequency is one accident per 1450 reactor years.
   - How to explain such a gap?

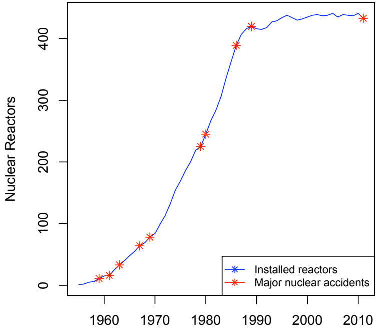
Data: Which events can be considered as nuclear accidents?

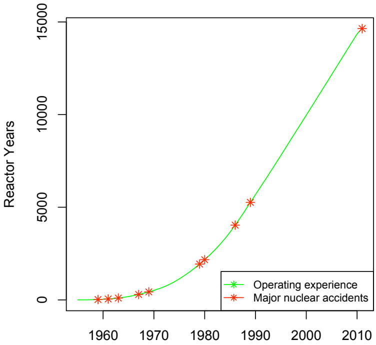
Table : Core melt downs from 1955 to 2011 in Cochran (2011)

Year	Location	Unit	Reactor type
1959	California, USA	Sodium reactor experiment	Sodium-cooled power reactor
1961	Idaho, USA	Stationary Low Reactor	Experimental gas-cooled, water moderated
1966	Michigan, USA	Enrico Fermi Unit 1	Liquid metal fast breeder reactor
1967	Dumfreshire, Scotland	Chapelcross Unit 2	Gas-cooled, graphite moderated
1969	Loir-et-Cher, France	Saint-Laurent A-1	Gas-cooled, graphite moderated
1979	Pennsylvania, USA	Three Mile Island	Pressurized Water Reactor (PWR)
1980	Loir-et-Cher, France	Saint-Laurent A-1	Gas-cooled, graphite moderated
1986	Pripyat, Ukraine	Chernobyl Unit 4	RBKM-1000
1989	Lubmin, Germany	Greifswald Unit 5	Pressurized Water Reactor (PWR)
2011	Fukushima, Japan	Fukushima Dai-ichi Unit 1,2,3	Boiling Water Reactor (BWR)

Core melt downs

We have focused dangerous events even if no radioactive material was released out of the unit





Findings

Table : The Fukushima Daiichi effect

Model	$\hat{\lambda}_{2010}$	$\hat{\lambda}_{2011}$	Δ
MLE Poisson	6.175e-04	6.66e-04	0.0790
Bayesian Poisson-Gamma	4.069e-04	4.39e-04	0.0809
Poisson with time trend	9.691e-06	3.20e-05	2.303
PEWMA	4.420e-05	1.95e-03	43.216

Bayesian Gamma-Poisson Model

This model is described by the following equations:

Poisson likelihood

$$f(y_t|\lambda) = \frac{(\lambda E_t)^{y_t} \exp(-\lambda E_t)}{y_t!} \quad (1)$$

Prior λ distribution

$$f_0(\lambda) = \frac{\exp(-b\lambda)\lambda^{a-1} b^a}{\Gamma(a)} \quad (2)$$

Formula to update the parameters

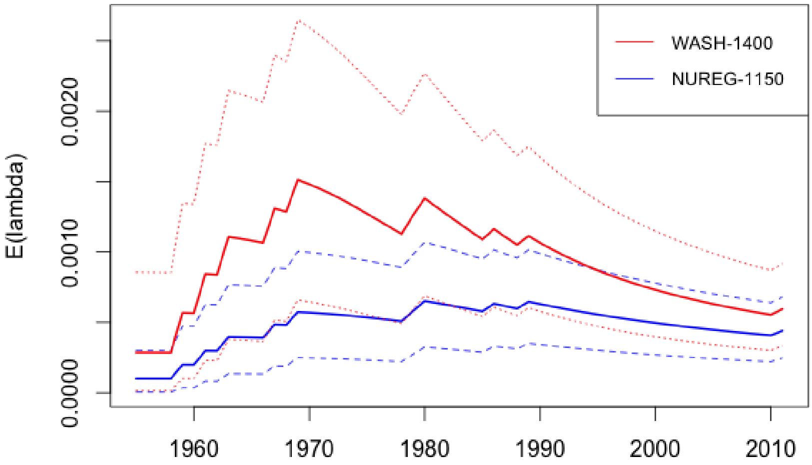
$$a_u = a + y_t \quad (3)$$

$$b_u = b + E_t \quad (4)$$

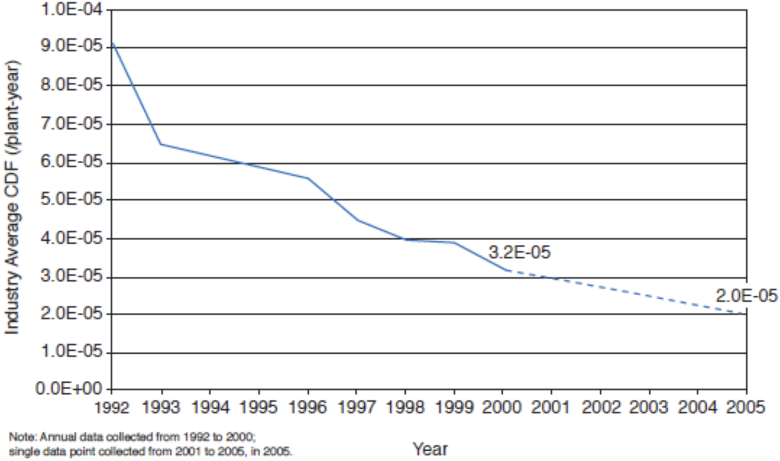
Which prior should we use?

- Probabilistic risk assessment (PRA) estimates have valuable information to construct the prior
- In the U.S nuclear fleet, this models are widely used because their results are a key input for the risk-based nuclear safety regulation approach
- The Nuclear Regulatory Commission has been doing PRA studies since 1975
  1. The oldest one is the WASH-1400 (1975)
     - This first study estimated a CDF equal to 5E-05 and suggested an upper bound of 3E-04
     - In terms of prior parameters this can be expressed as ($a = 1, b = 3500$)
  2. The updated version of the first PRA is in the NUREG 1150 (1990)
     - Computed a CDF equal to 8.91E-05
     - The prior would be ($a = 1, b = 10000$)

Bayesian Gamma-Poisson Model results



Safety improvements



Time-varying mean

Table : Poisson with deterministic time trend

Database	Coefficients	Estimate	Std.Error	Z value	$\Pr(> z)$	
CMD	Intercept	221.886	55.788	3.977	6.97e-05	***
	Time	-0.115	0.028	-4.091	4.29e-05	***

These results confirm that the arrival rate of a nuclear accident has changed along this period. However this finding challenge the underlying independence assumption of the Poisson model.

Independence and PEWMA model

- When we assume that we have an independent and identically distributed (i.i.d) sample, we give the same weight to each observation ⇒ In the Poisson regression all the accidents are equally important in the estimation
- But we have a time series, thus if events that we observe today are somehow correlated with those in the past ⇒ We should give more weight to recent events than those in the past.
- We propose to use a structural event-count time series model. This framework has been developed by Harvey and Fernandes (1989) and Brandt and Williams (1998)
- This model is called Poisson Exponentially Weighted Moving Average (PEWMA)

PEWMA model

PEWMA model has a time changing mean λ_t that is described by two components:

Observed component: Given by a log-link as in Poisson regression that contains the explanatory variables that we observe at time t. We are interested in knowing how these variables affect the current state, which are represented with β coefficients

Unobserved component: Shows how shocks persist in the series, therefore it captures data dependence across time. This dependence is represented by a smoothing parameter defined as ω

ω is the key parameter of PEWMA model, because it represents how we discount past observations in current state.

- If $\omega \rightarrow 0$ this means that the shocks persist in the series. We have high dependence in the data
- If $\omega \rightarrow 1$ this will indicate that events are independent

PEWMA Equations

Bayesian approach in PEWMA model

To find the density of the parameter across time we use a Kalman filter that recursively uses Bayesian updating. The procedure consists in combining a prior distribution $\Gamma(a_{t-1}, b_{t-1})$ with the transition equation to find $\pi(\lambda_t | Y_{t-1})$ that is a Gamma distribution

$$\lambda_t | Y_{t-1} \sim \Gamma(a_{t|t-1}, b_{t|t-1})$$

Where:

- $a_{t|t-1} = \omega a_{t-1}$
- $b_{t|t-1} = \omega b_{t-1} \frac{\exp(-X_t' \beta - r_t)}{E_t}$

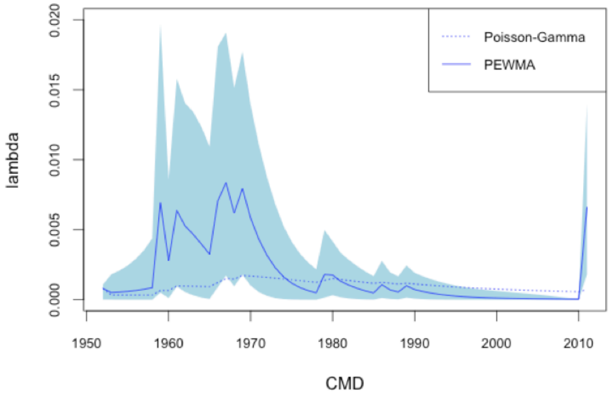
The Gamma parameters are updated following a simple formula:

- $a_t = a_{t|t-1} + y_t$
- $b_t = b_{t|t-1} + E_t$

PEWMA results and Fukushima Dai-ichi effect

Table : PEWMA Estimates for ω with a Prior a=1 b=3.500 (WASH-1400)

Database	Parameters	Std. Errors	Z-score
CMD	0.801	0.024	32.850



Fukushima effect

- With PEWMA model, the Fukushima Daiichi accident results in a huge increase in the arrival rate estimate (i.e x 50 times). Is this change unrealistic?
- No. This accident is not a black swan
  1. Seisms followed by a wave higher than 10 meters have been previously documented. This knowledge appeared in the 80s when the unit was already built but it has then been ignored by the operator.
  2. It has also been ignored by the nuclear safety agency NISA because as well-documented now the Nippon agency was captured by the nuclear operators (Gundersen (2012)).
- This accident revealed the risks associated with the NPPs in the world that have been built in hazardous areas and have not been retrofitted to take into account better information on natural risks collected after their construction
- It also revealed that even in developed countries NPP might be under-regulated by a non-independent and poorly equipped safety agency as NISA.

Conclusion

1. When it is assumed that the observations are independent the arrival of a new event (last nuclear catastrophe) did not increase the estimates. The Fukushima effect is close to 8% regardless of whether you use a basic Poisson or a Bayesian Poisson Gamma model
2. The introduction of a time-trend captures the effect of safety and technological improvements in the arrival rate.This model led to a significant increase in the arrival rate because of Fukushima Dai-ichi meltdowns.
3. The PEWMA model has allowed us to test the validity of the independence hypothesis and we find that it is not the case. This seems to be more suitable because recent events carry more information that those accidents in the past.
4. With PEWMA model the Fukushima Dai-ichi event represents a substantial, but not unrealistic increase in the estimated rate.

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Poisson Exponentially Weighted Moving Average (PEWMA)

1. Measurement equation: Is the stochastic component $f(y_t|\lambda_t)$, we keep our assumption that y_t is distributed Poisson with arrival rate λ_t

$$\lambda_t = \lambda^*_{t-1} \frac{\exp(X'_t \beta)}{E_t}$$

The rate has two components:

- An unobserved component λ^*_{t-1}
- A log-link like in Poisson regression

PEWMA Model equations

2. Transition equation: Shows how the mean changes over time.

$$\lambda_t = \lambda_{t-1} \exp(r_t)\eta_t$$

- r_t is the rate of growth
- η_t is a random shock that is distributed $B(a_{t-1}\omega, (1 - a_{t-1})\omega)$
- ω is weighting parameter. When $\omega \to 1$ observations are independent, $\omega \to 0$ the series is persistent

3. Prior distribution: Describes the initial state.

$$\lambda^*_{t-1} \sim \Gamma(a_{t-1}, b_{t-1})$$

- We are going to use the conjugate for the Poisson that a Gamma distribution

Kalman filter procedure

We are interested in finding:

$$f(y_t|Y_{t-1}) = \int_0^{\infty} \underbrace{f(y_t|\lambda_t)}_{Measurement} \underbrace{\pi(\lambda_t|Y_{t-1})}_{Unknown} d\lambda_t$$

So to find $\pi(\lambda_t|Y_{t-1})$ we use a Kalman filter. Following these steps:

1. Combine the prior distribution of λ_{t-1} with the transition equation to find the distribution of $\lambda_t|Y_{t-1}$
2. Using the properties of the gamma distribution we find the parameters $a_{t|t-1}, b_{t|t-1}$
3. We use the Bayes' updating formula to compute the distribution of $\lambda_t|Y_t$ whenever the information set is available (i.e $\forall t < T$)
4. This updated distribution becomes the prior in the next period and we repeat the previous steps

Posterior distribution for λ_t

When we combine the transition function with the prior is possible to show that:

$$\lambda_t | Y_{t-1} \sim \Gamma(a_{t|t-1}, b_{t|t-1})$$

Where:

- $a_{t|t-1} = \omega a_{t-1}$
- $b_{t|t-1} = \omega b_{t-1} \frac{\exp(-X_t' \beta - r_t)}{E_t}$

The posterior distribution is also Gamma and the parameters are updated following a simple formula:

$$\lambda_t | Y_t \sim \Gamma(a_t, b_t)$$

Where:

- $a_t = a_{t|t-1} + y_t$
- $b_t = b_{t|t-1} + E_t$

Log-likelihood function

Now we can compute $f(y_t|Y_{t-1})$ that is given by a negative binomial density function

$$
\begin{aligned}
f(y_t|Y_{t-1}) = & \int_0^\infty f(y_t|\lambda_t)\pi(\lambda_t|Y_{t-1})d\lambda_t \\
= & \frac{\Gamma(\omega a_{t-1}+y_t)}{y_t!\Gamma(\omega a_{t-1})}\{\omega b_{t-1}\frac{\exp(-X_t'\beta-r_t)}{E_t}\}^{\omega a_{t-1}} \\
& \times\{E_t+\omega b_{t-1}\frac{\exp(-X_t'\beta-r_t)}{E_t}\}^{-(\omega a_{t-1}+y_t)}
\end{aligned}
$$

So the predictive joint distribution is

$$f(y_0, ..., Y_T) = \prod_{t=0}^{T} f(y_t|Y_{t-1})$$

The log-likelihood function is based on this joint density

$$\mathcal{L} = \log(f(y_0, ..., Y_T))$$

Go back