The effects of oil price shocks in a new-Keynesian framework with capital accumulation

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Séminaire de recherches PSL en économie de l'énergie
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Purpose

- Recall the difference between oil's output elasticity and oil's cost share.

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  $\Rightarrow$ A theoretical study with DSGE model

  $\Rightarrow$ An empirical approach with U.S data (1984:Q1-2007:Q1)

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  $\Rightarrow$ An empirical approach with U.S data (1984:Q1-2007:Q1)

- Analyze the role and evolution of oil dependency.

Outline

The 1970s' oil shocks

	Year	Change
Real (2013-2014) Oil Price	1973 – 1974	+150%
	1978 – 1980	+100%
Inflation	1973–1974	+4.3 %
	1979–1980	+5.9 %
Unemployment rate	1973 – 1974	+3.6 points
	1979 – 1982	+3.8 points
Growth	1973–1975	-6%
	1979–1980	-5.8%
Real Wages	1973–1975	-2.7%
	1979–1980	-1.3%

Literature: The correlation between oil shocks and the business cycles

- Hamilton (1983, 1986,1989)
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A correlation **challenged** by:

- Bernanke, B., et al. (1997): The role of monetary policy.

The 2000s' oil shock

	Year	Change
Real (2013-2014) Oil Price	1973 – 1974	+150%
	1978 – 1980	+100%
	2002 – 2007	+147%
Inflation	1973–1974	+4.3 %
	1979–1980	+5.9 %
	2002-2007	+1.3%
Unemployment rate	1973 – 1974	+3.6 points
	1979 – 1982	+3.8 points
	2002-2007	+1.2 points
Growth	1973–1975	-6%
	1979–1980	-5.8%
	2002-2007	+2.7% (average)
Real Wages	1973–1975	-2.7%
	1979–1980	-1.3%
	2002-2005	-0.4%

The debate

Barsky & Kilian (2004)	Hamilton (2009)	Blanchard & Galí (2009) Blanchard & Riggi (2013)
overstated link between oil price changes and macroeconomic performance	different causes, but similar consequences	• the reduction of oil share in production; • the flexibilization of real wages and; • the improvements in monetary policy.

Research Questions

- Do we really understand how oil shocks spread in the economy?
- Is the U.S economy really invulnerable to oil shocks? If so, what change in the U.S to make the economy immune?
- What kind of policy could be implemented to help to lessen effects of oil shocks?

What have we done?

To the best of our knowledge, no dynamic general equilibrium model was available that captures the next two stylized facts:

1. The stagflationary impact of sharp oil real price rise.
2. The various impacts of capital accumulation:
   - Hysteresis effect
   - The potential role of capital as a new channel for monetary policy
   - The role of capital energy efficiency in dampening the impact of an oil price rise

What have we done?

The present paper introduces oil into a DSGE model in the same way as Blanchard & Galí (2009) and Blanchard & Riggi (2013), to which it adds capital accumulation.

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The oil's output elasticity

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The present paper introduces oil into a DSGE model in the same way as Blanchard & Galí (2009) and Blanchard & Riggi (2013), to which it adds capital accumulation.

The oil's output elasticity $\neq$ oil's cost share

Why add capital in the model?

1. More realistic.
2. More reliable empirical estimation.
3. Separate oil from other types of capital.

Decoupling the cost share from output elasticity

$$\max_x \; Y(x) - p \cdot x \tag{1}$$

leads to:

$$\varepsilon_i := \frac{x_i}{Y(x)} \times \frac{\partial Y}{\partial x_i}(x) = \frac{p_i x_i}{p \cdot x}$$

Decoupling the cost share from output elasticity

$$\max_{x} \; Y(x) - p \cdot x \quad (2)$$
$$\text{s.t.} \;\; f(x) = 0$$

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$$\lambda \to +\infty \Rightarrow \varepsilon_i \to 1$$

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$$\lambda \to +\infty \Rightarrow \varepsilon_i \to 1$$

ε may take any real value between $-\infty$ and $x_i p_i / x \cdot p$ whenever $0 < \lambda < (p \cdot x)\frac{\partial x_i}{\partial f(x)}$

Decoupling the cost share from output elasticity

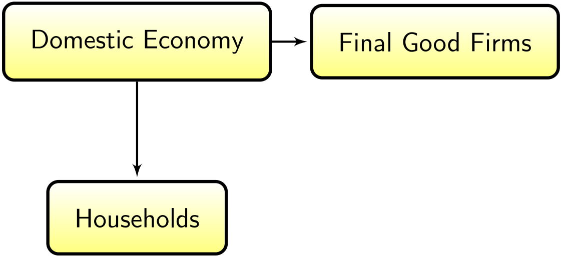
So that a large share $x_i p_i / x \cdot p$ is compatible with a small ε!

Outline

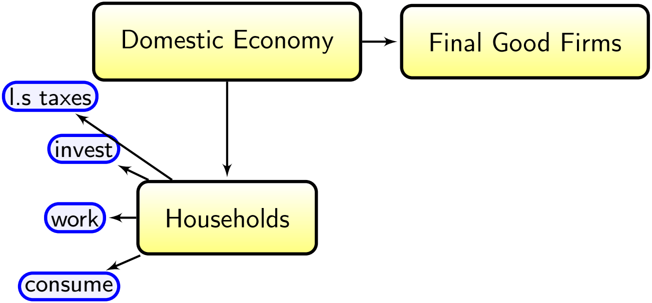
General Structure

Domestic Economy

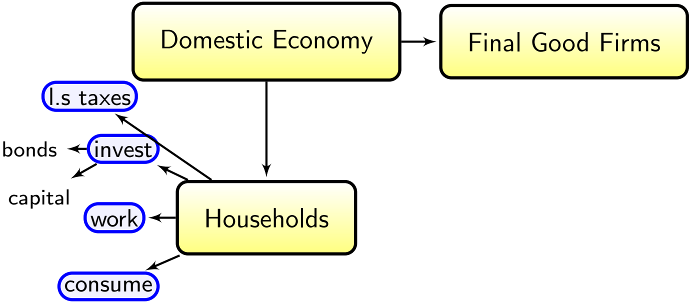
General Structure



General Structure



General Structure



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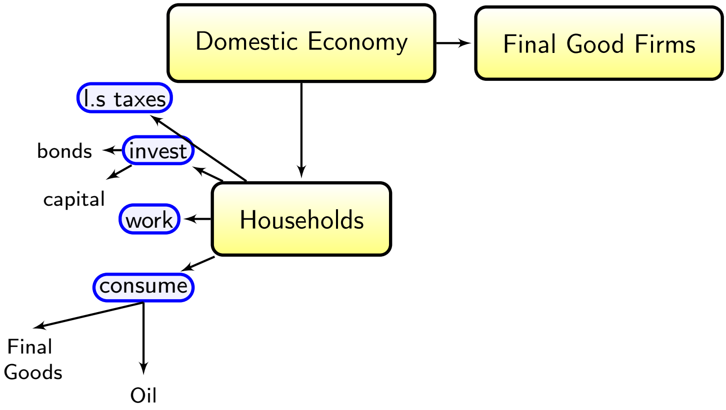

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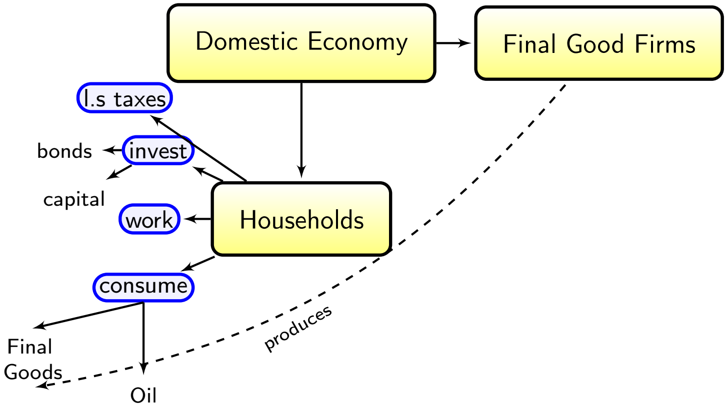

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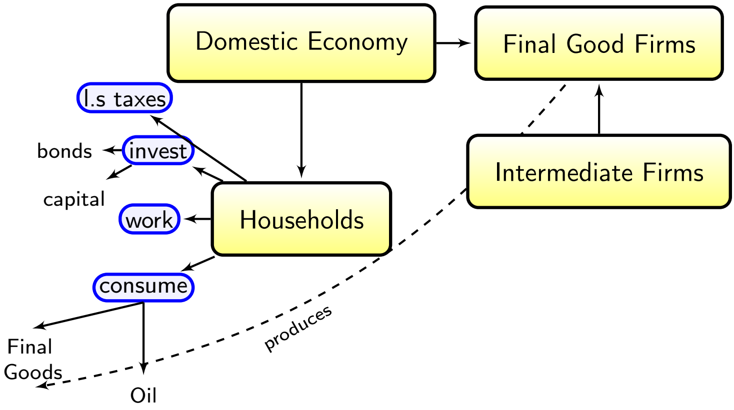

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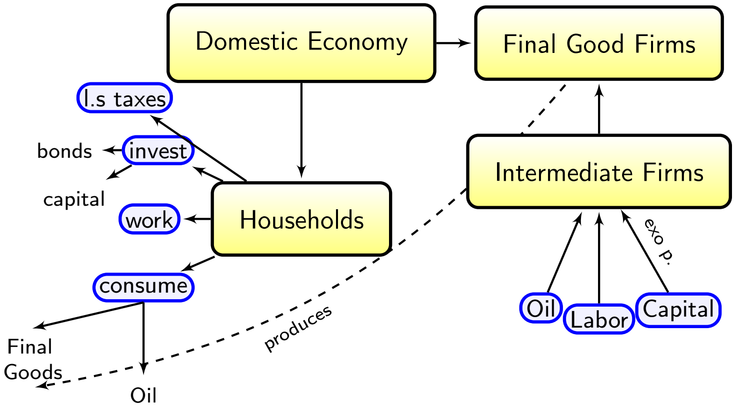

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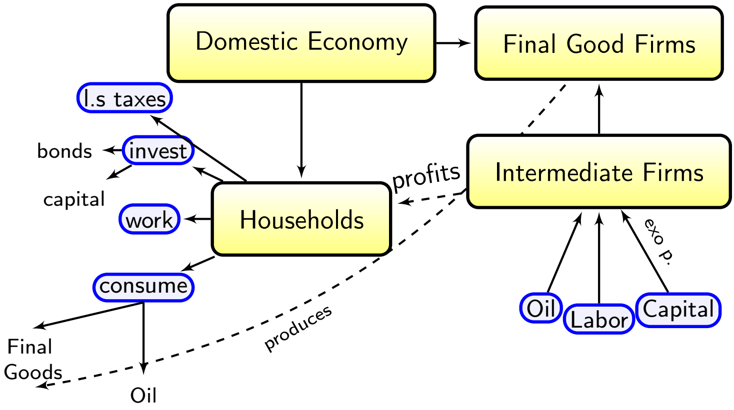
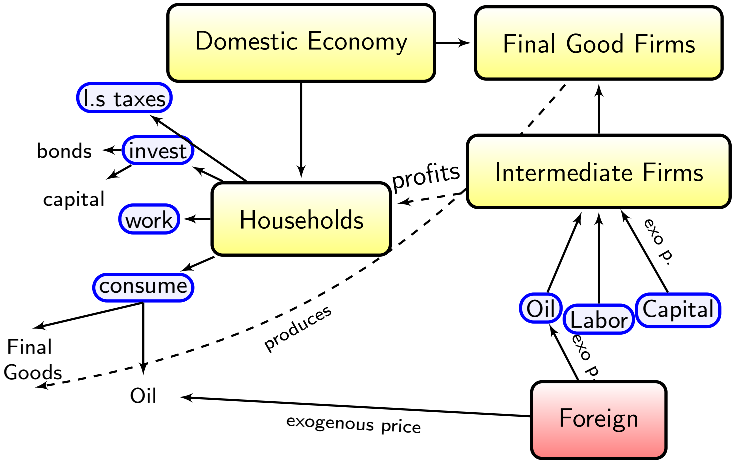
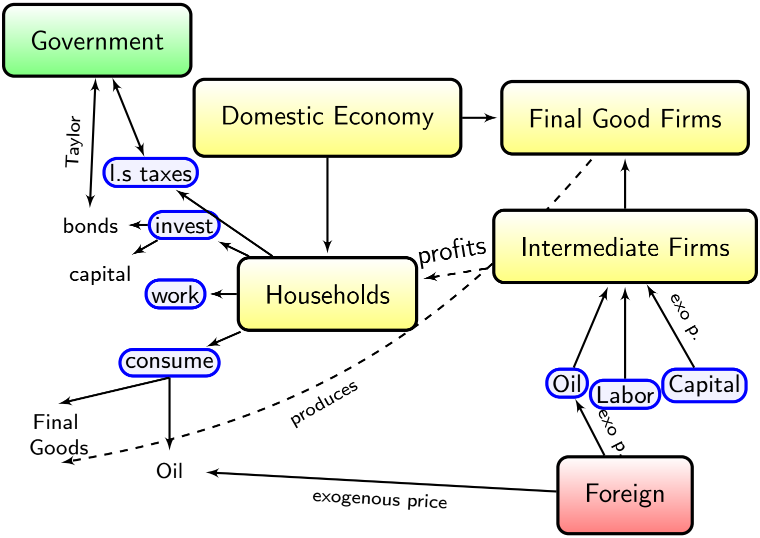


General Structure



General Structure



Households

Problem

$$\max_{C_t, L_t, B_t, K_{t+1}} \mathbb{E}_0 \left[\sum_{t=0}^{\infty} \beta^t U(C_t(j), L_t(j)) \right], \quad 0 < \beta < 1$$

s. t

$$P_{c,t} C_t(j) + P_{k,t} I_t(j) + B_t(j) \leq$$

$$(1 + i_{t-1}) B_{t-1}(j) + W_t(j) L_t(j) + D_t + r_t^k P_{k,t} K_t(j) + T_t$$

Households

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$$U(C_t(j), L_t(j)) = log(C_t(j)) - \frac{L_t(j)^{1+\phi}}{1+\phi}$$

s. t

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$$P_{c,t} C_t(j) + P_{k,t} I_t(j) + B_t(j) \leq$$
$$(1 + i_{t-1}) B_{t-1}(j) + W_t(j) L_t(j) + D_t + r_t^k P_{k,t} K_t(j) + T_t$$

$$I_t := K_{t+1} - (1 - \delta) K_t$$

Households

$$C_t(j) := \Theta_x C_{e,t}^{x}(j) C_{q,t}^{1-x}(j)$$

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↓

$$\Theta_x := x^{-x}(1-x)^{-(1-x)}$$

Households

$$C_{q,t}(j) := \left(\int_0^1 C_{q,t}(i,j)^{1-\frac{1}{\epsilon_p}} di\right)^{\frac{\epsilon_p}{\epsilon_p - 1}}$$

↑

$$C_t(j) := \Theta_x C_{e,t}^x(j) C_{q,t}^{1-x}(j)$$

↓

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Optimization

Household's Optimal Expenditure Allocation

Optimization

Household's Optimal Expenditure Allocation

↓

$$\max_{C_{q,t}(j), C_{e,t}(j)} P_{c,t} C_t(j)$$

s. t

$$P_{c,t} C_t(j) = P_{e,t} C_{e,t}(j) + P_{q,t} C_{q,t}(j)$$

$$C_t(j) := \Theta_x C(j)^{x}_{e,t} C(j)^{1-x}_{q,t}$$

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$$C_t(j) := \Theta_x C(j)^x_{e,t} C(j)^{\mathbf{1}-x}_{q,t}$$

$$P_{c,t} = P^x_{e,t} P^{\mathbf{1}-x}_{q,t}$$

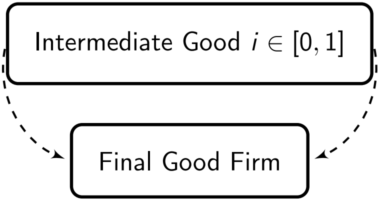
$$P_{q,t} C_{q,t}(j) = (1-x) P_{c,t} C_t(j)$$

$$P_{e,t} C_{e,t}(j) = x P_{c,t} C_t(j)$$

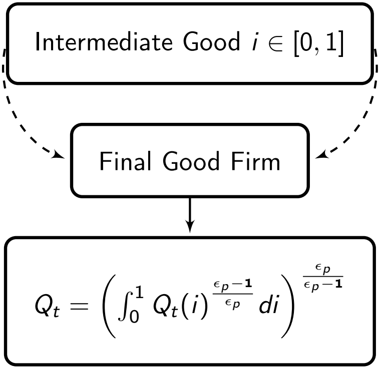
Final Good Producers

Final Good Firm

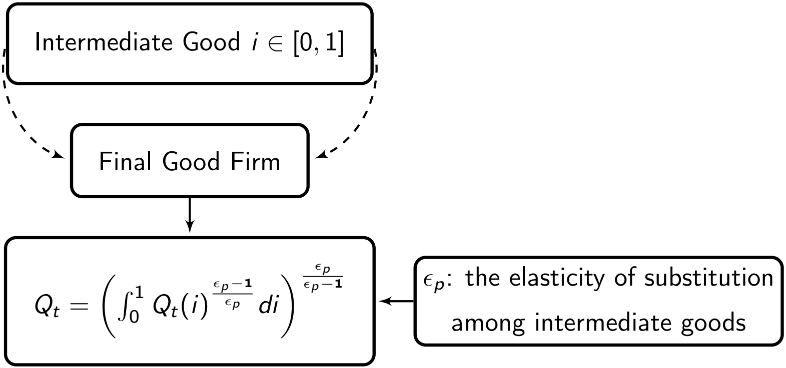
Final Good Producers



Final Good Producers



Final Good Producers



Final Good Producer Problem

Final Good Firm Profit Optimization

$$\max_{Q_t(i)} P_{q,t} Q_t - \int_0^1 P_{q,t}(i) Q_t(i) di$$

s. t

$$Q_t = \left(\int_0^1 Q_t(i)^{\frac{\epsilon_p - 1}{\epsilon_p}} di \right)^{\frac{\epsilon_p}{\epsilon_p - 1}}$$

i demand

$$Q_t(i) = \left(\frac{P_{q,t}(i)}{P_{q,t}} \right)^{-\epsilon_p} Q_t$$

final good price

$$P_{q,t} = \left(\int_0^1 P_{q,t}(i)^{1-\epsilon_p} di \right)^{\frac{1}{1-\epsilon_p}}$$

Intermediate Good Firms

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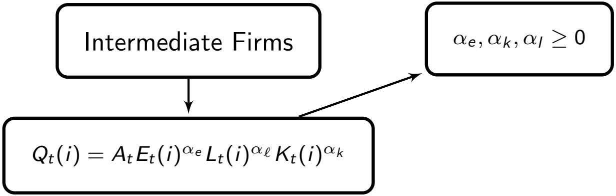
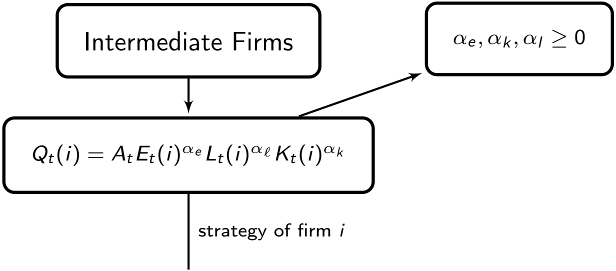
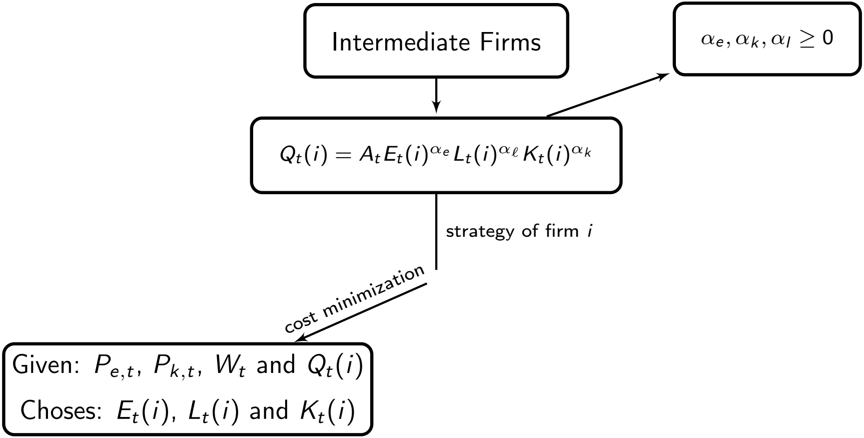


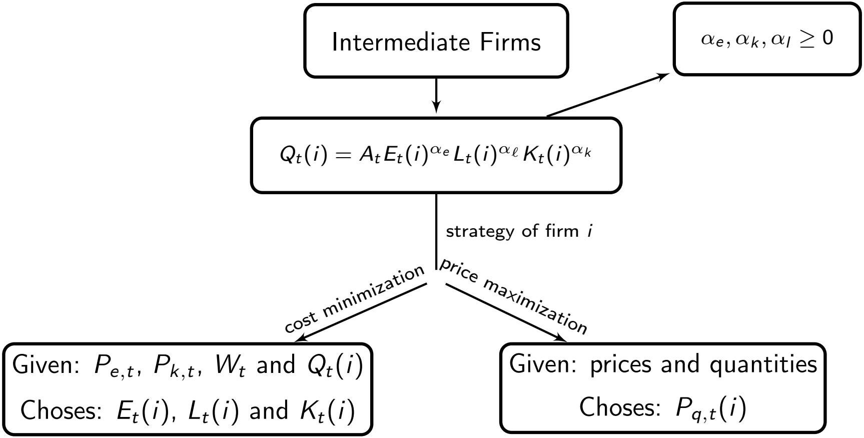
Intermediate Good Firms



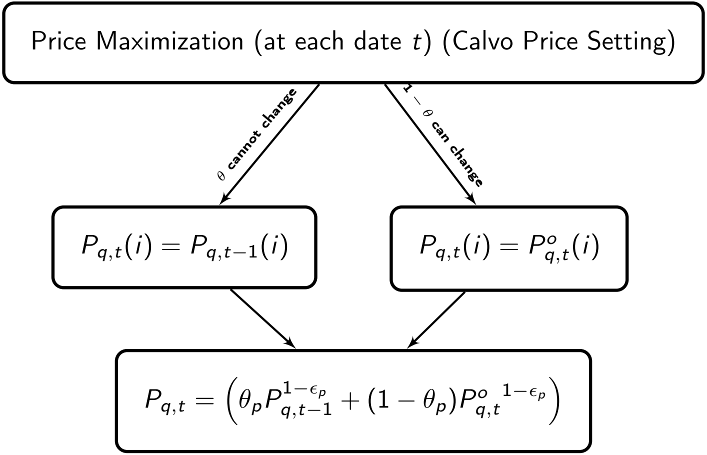
Intermediate Good Firms



Intermediate Good Firms



Price Optimization



GDP

GDP
(in value added)

↓

$$P_{c,t} Y_t = P_{q,t} Q_t - P_{e,t} E_t$$

GDP

GDP
(in value added)

$$P_{c,t} Y_t = P_{q,t} Q_t - P_{e,t} E_t$$

$$\alpha_e = \frac{\mathcal{M}_p * \text{Oil's Cost share}}{1 + \text{Oil's Cost share}}$$

GDP

Blanchard & Galí (2009) and Blanchard and Riggi (2013) define implicit GDP deflator ($P_{y,t}$) by:

$$P_{q,t} := P_{y,t}^{1-\alpha_e} P_{e,t}^{\alpha_e}$$

which yields to:

$$P_{y,t} = P_{q,t}^{\beta} P_{e,t}^{1-\beta}, \quad \beta > 1$$

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which yields to:

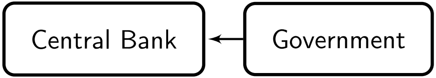
$$P_{y,t} = P_{q,t}^{\beta} P_{e,t}^{1-\beta}, \quad \beta > 1$$

We assume however that:

$$P_{y,t} = P_{c,t}$$

Government

Government



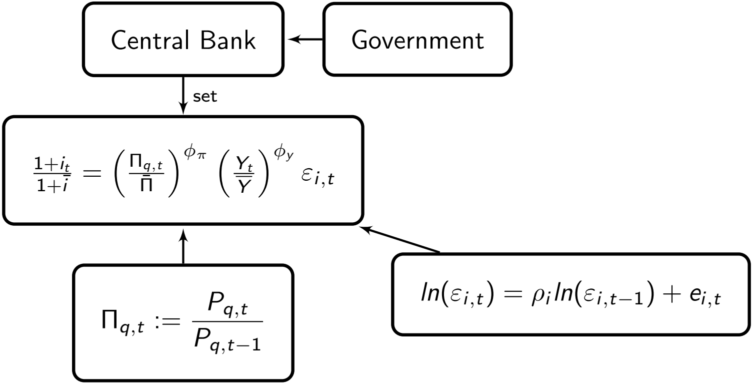
Government

Central Bank ← Government

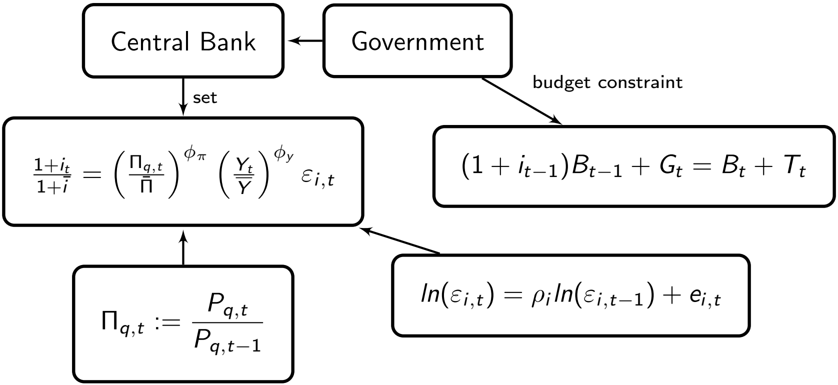
set

$$\frac{1+i_t}{1+\bar{i}} = \left(\frac{\Pi_{q,t}}{\bar{\Pi}}\right)^{\phi_\pi} \left(\frac{Y_t}{\bar{Y}}\right)^{\phi_y} \varepsilon_{i,t}$$

Government

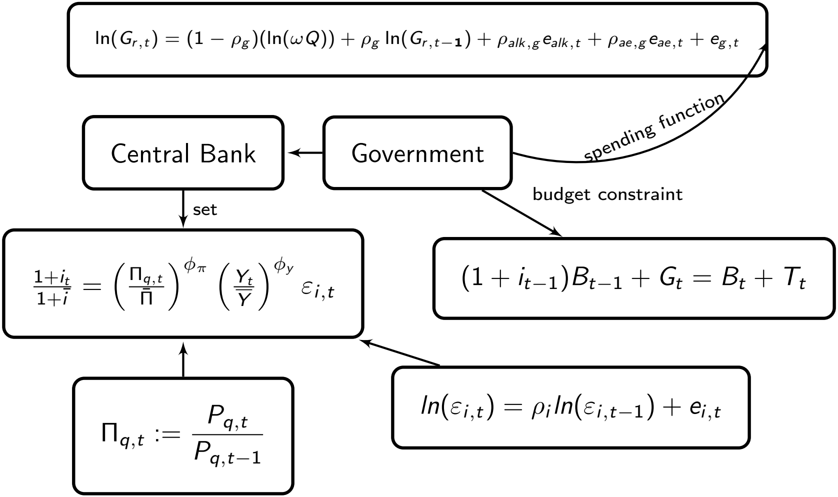

Central Bank
Government
set
$\frac{1+i_t}{1+\bar{i}} = \left(\frac{\Pi_{q,t}}{\bar{\Pi}}\right)^{\phi_\pi} \left(\frac{Y_t}{\bar{Y}}\right)^{\phi_y} \varepsilon_{i,t}$
$\Pi_{q,t} := \frac{P_{q,t}}{P_{q,t-1}}$
$ln(\varepsilon_{i,t}) = \rho_i ln(\varepsilon_{i,t-1}) + e_{i,t}$

Government



Central Bank
Government
set
budget constraint
$\frac{1+i_t}{1+\bar{i}} = \left(\frac{\Pi_{q,t}}{\bar{\Pi}}\right)^{\phi_\pi} \left(\frac{Y_t}{\bar{Y}}\right)^{\phi_y} \varepsilon_{i,t}$
$(1 + i_{t-1})B_{t-1} + G_t = B_t + T_t$
$\Pi_{q,t} := \frac{P_{q,t}}{P_{q,t-1}}$
$ln(\varepsilon_{i,t}) = \rho_i ln(\varepsilon_{i,t-1}) + e_{i,t}$

Government


$\ln(G_{r,t}) = (1-\rho_g)(\ln(\omega Q)) + \rho_g \ln(G_{r,t-1}) + \rho_{alk,g} e_{alk,t} + \rho_{ae,g} e_{ae,t} + e_{g,t}$
spending function
Central Bank
Government
budget constraint
set
$\frac{1+i_t}{1+\bar{i}} = \left(\frac{\Pi_{q,t}}{\bar{\Pi}}\right)^{\phi_\pi} \left(\frac{Y_t}{\bar{Y}}\right)^{\phi_y} \varepsilon_{i,t}$
$(1+i_{t-1})B_{t-1} + G_t = B_t + T_t$
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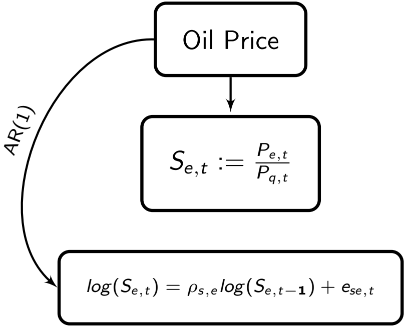
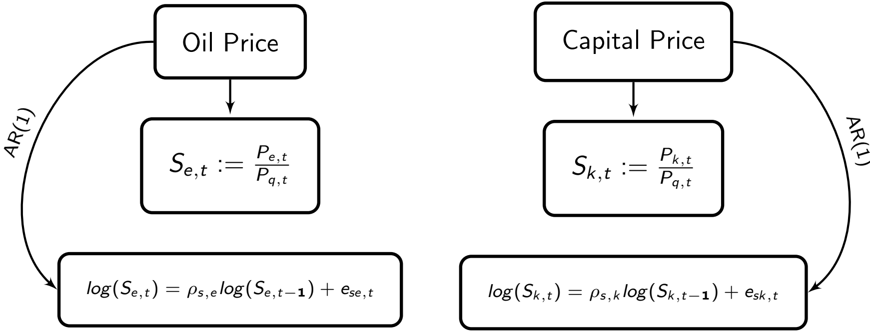
Shocks



Shocks


Oil Price
$S_{e,t} := \frac{P_{e,t}}{P_{q,t}}$
AR(1)
$log(S_{e,t}) = \rho_{s,e} log(S_{e,t-1}) + e_{se,t}$
Capital Price
$S_{k,t} := \frac{P_{k,t}}{P_{q,t}}$
AR(1)
$log(S_{k,t}) = \rho_{s,k} log(S_{k,t-1}) + e_{sk,t}$

Shocks

TFP

↓ AR(1)

$$ln(A_{LK,t}) = \rho_a ln(A_{LK,t-1}) + e_{alk,t}$$

Shocks

TFP

AR(1)

$$ln(A_{LK,t}) = \rho_a ln(A_{LK,t-1}) + e_{alk,t}$$

Price Markup

ARMA(1,1)

$$\varepsilon_{p,t} = \rho_p \varepsilon_{p,t-1} + e_{p,t} - \nu_p e_{p,t-1}$$

Outline

Data

1984:Q1–2007:Q1

Observed Variable	Transformation
labobs	$ln\left(\frac{Averagehours*CE16OVIndex}{LNSIndex}\right)*100-mean\left(ln\left(\frac{Averagehours*CE16OVIndex}{LNSIndex}\right)*100\right)$
infobs	$ln\left(\frac{GDPDEF}{GDPDEF(-1)}\right)*100-mean\left(ln\left(\frac{GDPDEF}{GDPDEF(-1)}\right)*100\right)$
iobs	$\left(ln\left(1+\frac{FEDFUND}{400}\right)-mean\left(ln\left(1+\frac{FEDFUND}{400}\right)\right)\right)*100$
eobs	$ln\left(\frac{TotalSAOil}{LNSIndex}\right)*100-mean\left(ln\left(\frac{TotalSAOil}{LNSIndex}\right)*100\right)$
invobs	$detrend\left(ln\left(\frac{\frac{PFI}{GDPDEF}}{LNSIndex}\right)*100\right)$
yobs	$detrend\left(ln\left(\frac{GDPC09}{LNSIndex}\right)*100\right)$

Calibrated Parameters

β	δ	ϵ_p	ω	x
0.99	0.025	8	0.18	0.023

Table: Calibrated Parameters

Identification Analysis

Lack of consensus over the value of oil's output elasticity.

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Lack of consensus over the value of oil's output elasticity.

$\Rightarrow$ we perform an identification analysis

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Result:

Identification Analysis

Lack of consensus over the value of oil's output elasticity.

$\Rightarrow$ we perform an identification analysis

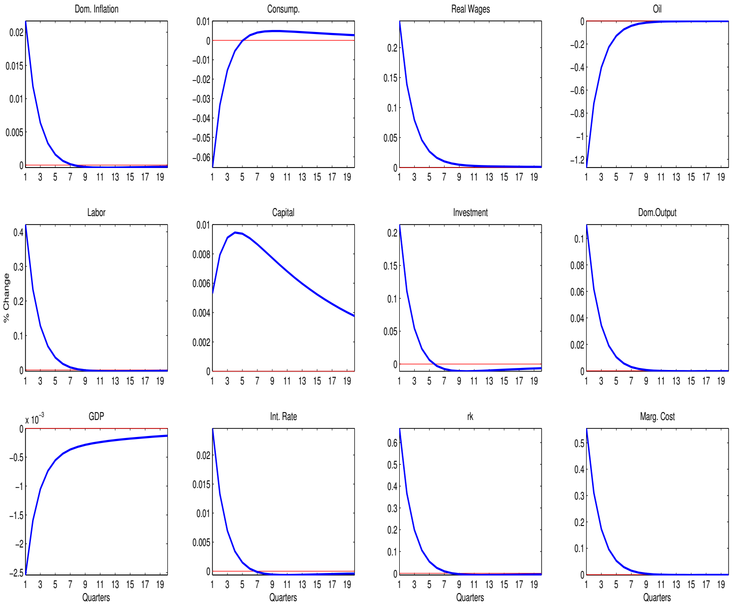
Result:

If the chosen prior for the output elasticity parameter is high, the price Calvo parameter looses identification strength.

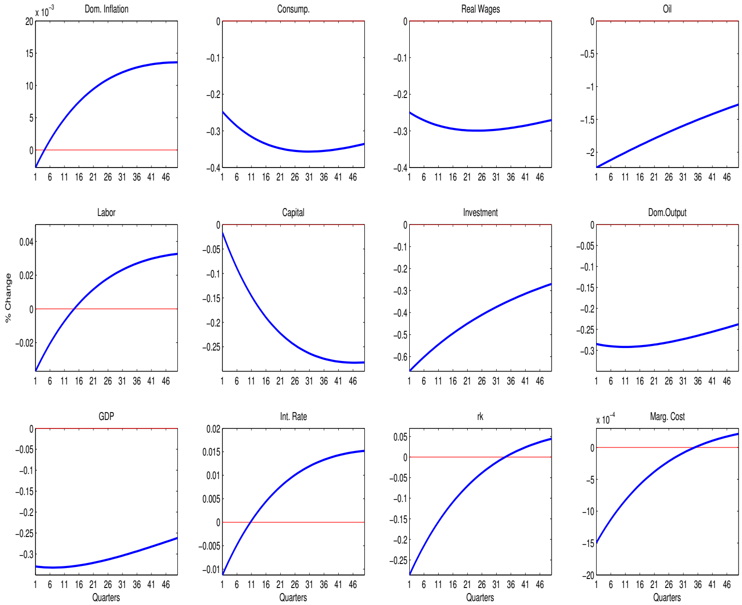
Table: Prior and Posterior Distribution of Structural Parameters

Parameter		Prior distribution	Posterior distribution			
			Mode	Mean	10%	90%
θ estimated						
Capital elasticity	α_k	IGamma(0.1,2)	0.3728	0.3599	0.3380	0.3822
Labor elasticity	α_ℓ	IGamma(0.4,2)	0.6424	0.6411	0.6111	0.6745
Oil elasticity	α_e	IGamma(0.6,2)	0.1234	0.1254	0.1051	0.1460
Inverse Frisch elasticity	ϕ	IGamma(1.17,0.5)	0.6209	0.6308	0.4736	0.8019
Taylor rule response to inflation	ϕ_π	Normal(1.2,0.1)	1.2235	1.2253	1.0686	1.3558
Taylor rule response to output	ϕ_y	Normal(0.5,0.1)	0.8020	0.7882	0.6884	0.8876
Calvo price parameter	θ	Beta(0.5,0.1)	0.9812	0.9812	0.9380	0.9883
θ calibrated						
Capital elasticity	α_k	IGamma(0.2,2)	0.3918	0.3809	0.3624	0.3989
Labor elasticity	α_ℓ	IGamma(0.4,2)	0.5947	0.5966	0.5622	0.6305
Oil elasticity	α_e	IGamma(0.5,2)	0.1132	0.1177	0.0915	0.1434
Inverse Frisch elasticity	ϕ	IGamma(1.17,0.5)	1.2562	1.2625	0.9073	1.6069
Taylor rule response to inflation	ϕ_π	Normal(1.2,0.1)	1.5236	1.5307	1.3883	1.6722
Taylor rule response to output	ϕ_y	Normal(0.5,0.1)	0.0265	0.0214	0.0001	0.0402

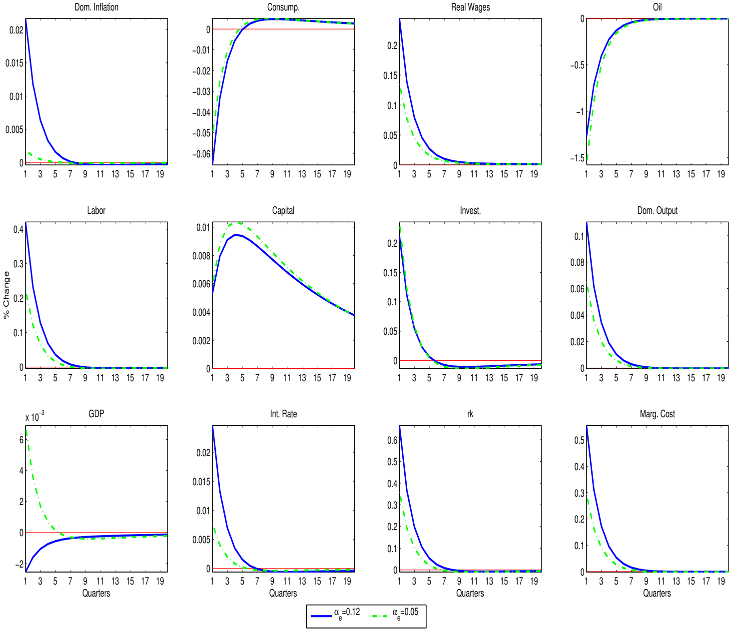
Outline



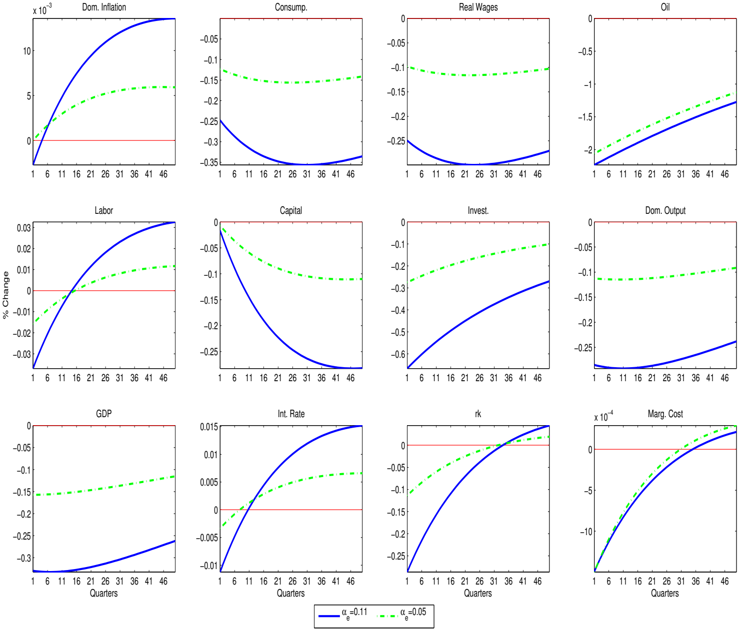
Response to one Standard Deviation Shock (1.94 %) on Real Price of Oil. Case: θ estimated



Response to one Standard Deviation Shock (1.94 %) on Real Price of Oil. Case: θ calibrated

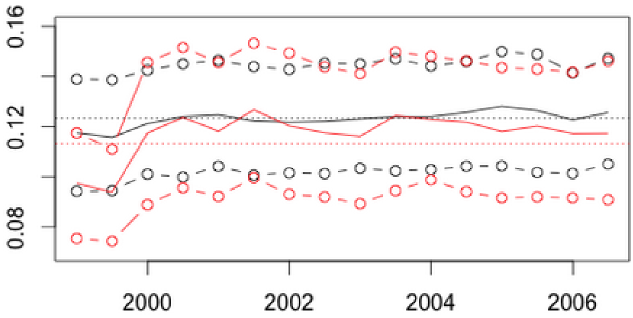


The Ecological Transition Effect. Case: θ Estimated

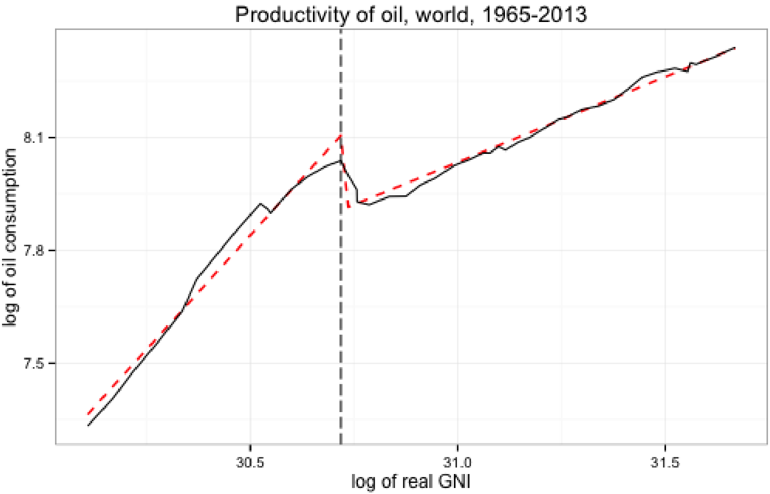


The Ecological Transition Effect. Case: θ Calibrated

The Evolution of $\hat{\alpha}_e$ from 1999:Q1 to 2006:Q3 in Bi-annual Frequency



The reduction of oil's dependence



Outline

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  ⇒ Reducing the output elasticity of oil is a promising policy recommendation.

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⇒ The 1979's oil productivity increase explains **in part** the difference of oil shocks between 2000s' and the one in 1970s'.

- **However**, there is no empirical evidence that this has been the case in the 2000s'.

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- **Increasing** aggregate returns to scale.
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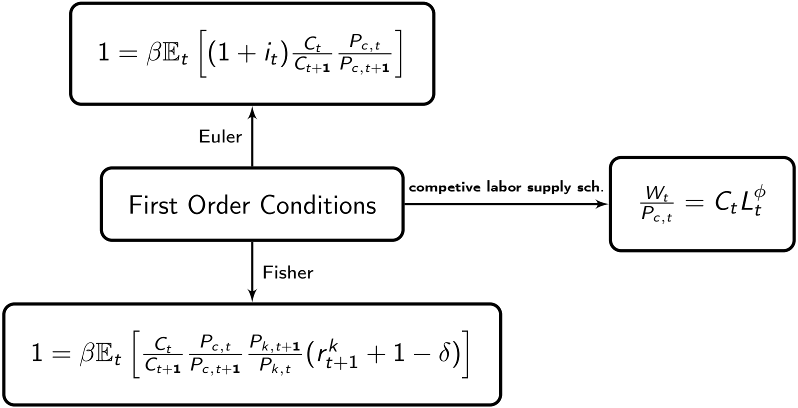
  Oil's output elasticity **is larger than** the oil's cost share value.

$\Rightarrow$ Oil's cost share and oil's output elasticity **are not** necessarily equal.

Thank you for your attention!

Household's Optimization



Cost Minimization

Cost minimization

↓ F.O.C

$$MC_t = \frac{W_t}{\alpha_\ell \frac{Q_t(i)}{L_t(i)}} = \frac{r_t^k P_{k,t}}{\alpha_k \frac{Q_t(i)}{K_t(i)}} = \frac{P_{e,t}}{\alpha_e \frac{Q_t(i)}{E_t(i)}}$$

↓

$$cost(Q_t(i)) = \alpha F_t Q_{(} i)^{\frac{1}{\alpha}}$$

Calvo Price Setting

Calvo Price Setting Problem

$$\max_{P_{q,t}(i)} \mathbb{E}_t \left[\sum_{k=0}^{\infty} \theta^k d_{t,t+k} \left[P_{q,t}(i) Q_{t,t+k}(i) - cost(Q_{t,t+k}(i)) \right] \right]$$

s.t

$$Q_{t,t+k}(i) = \left(\frac{P_{q,t}(i)}{P_{q,t+k}} \right)^{-\epsilon} Q_{t+k}, \quad \forall k \geq 0$$

Calvo Price Setting

Calvo Price Setting Solution

$$\mathbb{E}_t \left[\sum_{k=0}^{\infty} \theta_p^k d_{t,t+k} Q^o_{t+k|t} \left(P^o_{q,t} - \mathcal{M}^p mc^o_{t+k|t} \right) \right] = 0$$

$$d_{t,t+k}(j) := \beta^k \frac{\lambda_{t+k}(j)}{\lambda_t(j)}$$

$$MC^o_{t+k|t} := MC_{t+k}$$

$$Q^o_{t+k|t} := \left(\frac{P^o_{q,t}}{P_{q,t+k}} \right)^{-\epsilon_p} Q_{t+k}$$

Definition of Equilibrium

Equilibrium

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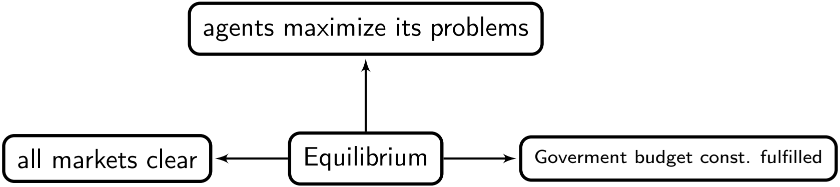

No Ponzi Scheme

Transversality condition (no Ponzi Scheme)

$$\lim_{k \to \infty} \mathbb{E}_t \left(\frac{B_{t+k}}{\prod_{s=0}^{t+k-1} (1 + i_{s-1})} \right) \geq 0, \quad \forall t.$$