Green Technology Adoption and the Business Cycle

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Abstract

We analyze the adoption of green technology in a dynamic economy affected by random shocks. Uncertainty and agents' beliefs have an effect on private investment decisions. We characterize the impact of economic uncertainty on the expected time and on the likelihood to reach a targeted level of environmental quality, to avoid an environmental catastrophe, and we appraise the costs of these shocks in terms of delays, policy readjustments, and welfare losses.

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1 Introduction

The increasing number of environmental issues that the world is facing has triggered a wide debate on how to switch toward sustainable development paths. Adoption of green technologies (AGT) is one amongst the main channels through which countries will be able to avoid environmental disasters without harming too much their well-being. However, in addition to the high

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level of uncertainty that governs most environmental issues, adopting new, cleaner technologies is risky for firms. At first, they incur a switching cost: green technologies are often more expensive, less productive, the workforce may not have the skills to operate the new technology, etc. Moreover, in the long run, investment choices may reveal to be inefficient, harming the firms' profitability. Network externalities and technological spillovers play an important role in determining what is the optimal technology that firms must adopt.[1] Public policies trigger firms to overcome this risk and to invest in green technologies, but the extent of this investment effort also depends on the prevailing economic environment. If the economy is facing a recession, available funds may be scarce, leading firms to postpone investment projects. We may thus expect that the path toward a sustainable growth will be stochastically affected by the same economic shocks that generate the business cycle.

In this paper we show that the high volatility in the adoption path of technology may lead to the failure of an optimally designed environmental policy if the dynamics of the environment are subject to irreversible tipping points, also called "environmental disasters". In our model, the industrial production continuously harms the environment which has an intrinsic ability to partly regenerate and recover from past damages. By investing in less polluting technologies, firms contribute to lowering their impact on the environment. We analyze the dynamic of this economy using a simple AGT model, focusing on an environmental policy that takes the form of subsidizing investment in green technologies. In our model, firms' profit depends on their "technology mix", a parameter that measures the polluting intensity of their production process. A key feature of our framework is that the optimal technology mix is neither the most recent one, nor the state of the art. Instead, it depends on the economic environment that firms face each period, i.e., a set of variables that may influence the profitability of the various production processes. Firms whose indexes are close to the current "optimal technology mix"are the most profitable ones.[2] In the absence of environmental regula-

[1] Consider for example that a firm decides to replace its fleet of fuel vehicles by electric ones (EV). If many firms expect EV to be used nationwide in the near future and decide to do the same, it is likely that the network of EV charging stations spreads broadly and using EV may become very convenient and cheap. If instead the rest of the economy gambles on hydrogen vehicles, then the few firms that have chosen EV may end-up being penalized.

[2] Firms using old and backward machines loose profit opportunities because their pro-

tion, this optimal green technology parameter evolves with the diffusion of knowledge, and depends on the average of the investment choices that firms have made.

By devoting a certain amount of resources to match the future optimal AGT index, firms draw a new technology parameter from a distribution that depends on their initial technology level and on their investment spending. This reflects the fact that a new machine or production process could perform more or less efficiently depending on the firm's other equipment and workforce's skills. As a result, the industrial sector is composed of a diverse set of firms. This diversity is also induced by the imperfect assessment firms have of the future economic environment. To decide on their investment, they need to form expectations about the future optimal technology mix. Their beliefs are based on private and public information that they aggregate in a Bayesian way. Firms may thus undershoot or overshoot the optimal AGT index.[3]

Because individuals do not consider the environmental footprint of the economy when making their choices, the social planner designs a policy that redirects the firms' technology indexes toward greener mixes. The policy implemented by the social planner internalizes the marginal benefit of AGT and thus of a better environmental quality on the welfare. However, it does not account explicitly for the probability of occurrence of an environmental disaster: the expected path of the environmental quality index (EQ) may approach some "tipping point"that triggers dire and irreversible consequences if the price assigned to the environmental quality is too low. Indeed, since business cycles and uncertainty affecting AGT make the path of the economy stochastic, the economy may at some point hit or even go below the tipping point while transiting toward a long term desirable value. A sustainable path thus corresponds to an appropriate valuation of the environmental quality, a price that offers a safety cushion to society: it must be such that, at a given confidence level, the environmental catastrophe is avoided.

Our model allows us to estimate confidence intervals for the realized paths

ductivity is low. But on the other hand, too advanced machines can be detrimental as well. They could require high skilled workforce to operate them, material employed may be difficult and costly to acquire, and they may involve large maintenance costs.

[3]The literature on global games and information aggregation (see, e.g. Morris & Shin, 2002, Angeletos & Werning, 2006) also considers, though in a static framework, agents whose payoffs from their investments are interdependent (they have a coordination motive) and have imperfect information an economic fundamentals.

of AGT and EQ indexes. We can therefore assess the risk of failure of the environmental policy (because of the occurrence of an environmental disaster), under various ranges of parameters. It also allows us to estimate the value environmental quality (the carbon price) that ensures that an environmental disaster is avoided with at a certain level of confidence. This is a novel and we believe, enriching approach on both the relative price of the environment and the optimal policy characterization. In the few integrated assessment models encompassing economic risk such as Golosov et al. (2014) or Traeger (2015), the optimal policy is derived given an exogenous marginal rate of substitution between consumption and environment. Moreover, there is no absorbing lower bound in the dynamics of the environmental quality, so that even though there is a risk that the environmental policy is not optimal given the realized shocks, there will be no irreversible consequences. Our model instead gives us a simple tool to derive the carbon price such that given the optimal green technology adoption subsidy, the environmental quality avoids hitting a tipping point with a large enough probability.

The literature displaying endogenous green growth focuses on productivity improvements and frontier innovation. This is the case in the AK paradigm where capital-knowledge accumulates by learning by doing (Stockey, 1998), in Lucas-like extensions (Bovenberg & Smulders, 1995), in a product variety framework (Gerlagh & Kuik, 2007) or in the Schumpetarian growth paradigm of destructive creation and directed technical changes (Acemoglu et al., 2012), where new innovations are adopted by markets as soon as they are invented. While frontier innovation is needed to make green technologies competitive compared to 'brown' ones, our focus is on adoption of existing green technologies that operates through diffusion of green inventions and a gradual replacement of the stock of old and polluting machines by greener ones. Our approach is close to the literature on endogenous growth viewed as a process of adoption of existing ideas and mutual imitation between firms, as exposed by Eaton & Kortum (1999); Lucas (2009); Lucas & Moll (2014); Perla & Tonetti (2014). In these papers, it is assumed that each agent in the economy is endowed with a certain amount of knowledge ("ideas") and this knowledge evolves through contact with the rest of the population. We adopt a similar approach to describe AGT: the R&D sector in our model is described as a pool of existing potential technologies, and the distribution of technology used amongst firms shifts over time according to the firms incentives to adopt new techniques.

Most of the literature on sustainable growth focuses on environmental uncertainty, that is, uncertainty in the frequency of catastrophic environmental events, on the size of the damage, or on the ability of the environment to recover from pollution (see, e.g. Tsur & Zemel, 1998, or De Zeeuw & Zemel, 2012, for analysis without AGT, and Bretschger & Vinogradova, 2014, and Soretz, 2007, for studies on the sharing of investment between productive capital and abatement measures in a AK setting.) Instead, we focus on shocks that affect the economy (like changes in international prices, demands shocks, etc.) and generate the so-called business cycle.

A few papers describe the responsiveness of optimal abatement policy to business cycles (Jensen & Traeger, 2014), or assess the optimal policy instrument in stochastic environments (Heutel, 2012; Fischer & Heutel, 2013) but they do not consider AGT.

The remaining of the paper is organized as follows: In section 2, we describe the economy and the dynamics of its main variables. In section 3, we characterize the first-best and derive the optimal policy to be implemented by a social planner. In section 4, we derive the economic forecasts allowed by our model. Section 5 is devoted to numerical simulations. The last section concludes.

2 Green technology dynamics

The economy is composed of a continuum of firms, of total mass equal to one, that collectively produce at date t an amount q_t of output, taken as the numeraire, which corresponds to the GDP of the economy. In the following, we abstract from the problem of production per se (in particular, the demand and supply of labor) to focus on the cleanliness of the production processes, i.e. the extent to which firms harm the environment while producing. We thus suppose that the dynamic of the GDP is given, and more specifically that it follows the following first-order autoregressive dynamic

$$q_t = gq_{t-1} + g_0 + \kappa_t, \tag{1}$$

with $g \geq 0$, $g_0 \geq 0$ and where κ_t corresponds to exogenous shocks that affect the economy at date t and is the realization of random variable $\tilde{\kappa}_t \sim \mathcal{N}(0, \sigma_\kappa^2)$.[4] We assume that $g < 1$, which implies that the per period expected

[4] Throughout the paper, a random variable is topped with symbol tilde ("˜") to distinguish it from its possible values.

increase in GDP, $g_0 - (1-g)q_{t-1}$, diminishes over time and converges to $q_S = g_0/(1-g)$.[5] The production processes used by firms are diverse and in the following, we suppose that the adequation of firm i's industrial production process to its economic environment at date t is captured by a real valued parameter x_{it} dubbed 'green technology index', and leads to profit

$$\pi(x_{it}, I_{it}; x_t^\star) = \Pi(x_t^\star) - (x_{it} - x_t^\star)^2/2 - I_{it} \tag{2}$$

where I_{it} is the investment the firm can make to improve its technology. Here, $x_t^\star$ represents the technology mix (of 'green' and 'brown' productive capital) that is the most efficient at time t, leading to profit level $\Pi(x_t^\star)$, and $(x_{it} - x_t^\star)^2/2$ the profit loss incurred by firm i due to a less effective technology, which depends on the distance between the firm's technology parameter x_{it} and this optimal value. This discrepancy is due to the quantities and prices of the specific inputs used by firm i, technical difficulties related to its workforce's skills (the employees' knowledge of the technology and their easiness to perform their tasks with a given technology), but also public policies like environmental taxes on polluting emissions that are more or less stringent depending on the cleanliness of the production process. Exceeding the optimal level $x_t^\star$ is costly because of the skills required to operate and maintain high-end technologies, the readiness (more or less immediate availability) of inputs employed and the cost of service they require. On the other end, less efficient technologies can be less productive and/or less environmental friendly, requiring costly end-of-pipe equipment to abide to environmental regulations (or to diminish their otherwise large environmental tax payments). On average, this leads to an AGT index of the industrial sector, $\mu_t \equiv \int x_{it} di$, that may be higher or lower than $x_t^\star$ depending of the investment policies of firms. Investment I_{it} allows the firm to adapt its technology to the economic environment that will prevail the next period, by buying new equipment or, for large investment, by replacing an entire production line. It is an incremental adaptation of the technology that may perform more or less efficiently depending on the existing equipment (machines that are not discarded) and the capability of the firm's employees to adapt to the new machinery. Formally, it corresponds for the firm to a draw

[5] The case $g > 1$ corresponds to a steady growth in economic wealth. Sustainable states in that case are not stationary: the economy should instead follow a sustainable balanced path.

of a new adequation level centered around $x_{it} + I_{it}$, i.e.

$$\tilde{x}_{it+1} = x_{it} + I_{it} + \tilde{\varepsilon}_{it} \tag{3}$$

where $\tilde{\varepsilon}_{it} \sim \mathcal{N}(0, \sigma_\varepsilon^2)$ is independent from κ_t for all t, with $\mathbb{E}\left[\tilde{\varepsilon}_{it}\tilde{\varepsilon}_{jt}\right] = 0$ for all i, j and $\int \tilde{\varepsilon}_{it} di = 0$. The firm chooses this investment level depending on its beliefs on the future values of the optimal technology parameter. The most efficient technology mix evolves with the diffusion of knowledge and the know-how of workers (the so-called knowledge spillover: the more firms invest, the better the workers' knowledge of new technologies in general) and network externality. We thus consider that the optimal technology mix of period $t+1$, $x^\star_{t+1}$, increases with total investment made in period t, according to

$$\tilde{x}^\star_{t+1} = \mu_t + \lambda \int I_{it} di + z_t + \tilde{\varepsilon}^\star_t, \tag{4}$$

where parameter $\lambda \in [0, 1)$ captures the spillovers due to firms' total investment, z_t is the effect of the environmental policy as chosen by the government to stimulate firms to invest in green technologies and $\tilde{\varepsilon}^\star_t \sim \mathcal{N}(0, \sigma_\star^2)$ a noise related to exogenous market conditions independent from $\tilde{\kappa}_t$. Using (3) to obtain $\int I_{it} di = \mu_{t+1} - \mu_t$, we get from (4) that

$$\tilde{x}^\star_{t+1} = \lambda \mu_{t+1} + (1 - \lambda)\mu_t + z_t + \tilde{\varepsilon}^\star_t. \tag{5}$$

Observe that absent governmental incentives ($z_t = 0$) and even if investment in green technology was positive in period t ($\mu_{t+1} > \mu_t$), the expected next period ideal mix is lower than the average mix μ_{t+1}. We thus assume that knowledge spillovers are not sufficient to spur investment in green technology under laissez-faire.

Firms form anticipations on the efficient technology mix one period to the next thanks to a public and firms' idiosyncratic signals, $\tilde{\omega}_t = x^\star_{t+1} + \tilde{\eta}_t$, and $\tilde{w}_{it} = x^\star_{t+1} + \tilde{\nu}_{it}$ respectively, where $\tilde{\eta}_t$ and $\tilde{\nu}_{it}$ are independently and normally distributed noises with zero mean and precision $\tau_\eta = \sigma_\eta^{-2}$ and $\tau_\nu = \sigma_\nu^{-2}$, with $\mathbb{E}[\tilde{\nu}_{it}\tilde{\varepsilon}^\star_t] = \mathbb{E}[\tilde{\eta}_t\tilde{\varepsilon}^\star_t] = 0$, the idiosyncratic noises also verifying $\mathbb{E}[\tilde{\nu}_{it}\tilde{\nu}_{jt}] = 0$ for all i, j and $\int \nu_{it} di = 0$.[6]Bayesian updating implies that posterior beliefs

[6]Hence, public and private signals allow firms to (imperfectly) coordinate their investment levels albeit their decisions are taken independently (in particular, investment plans are not best-reply functions). This dynamic global game between firms is solved sequentially with firms choosing each period their investment levels independently given their current anticipations of the next period ideal technology mix.

about $x^\star_{t+1}$ are normally distributed with mean

$$\hat{x}_{it+1} \equiv \mathbb{E}\left[\tilde{x}^\star_{t+1}|\omega_t, w_{it}\right] = \frac{\tau_\eta \omega_t + \tau_\nu w_{it}}{\tau_\eta + \tau_\nu} = x^\star_{t+1} + \tau\eta_t + (1-\tau)\nu_{it}, \quad (6)$$

for firm i, and variance

$$\hat{\sigma}^2_{it} = (\tau_\eta + \tau_\nu)^{-1}, \quad (7)$$

where $\tau = \tau_\eta/(\tau_\eta + \tau_\nu)$ is the relative precision of the public signal. Applying the principle of optimality, the firm's optimal investment plan is derived by using the Bellman equation which at date t is given by

$$\mathcal{V}(x_{it}; x^\star_t) = \max_{I_{it}} \pi(x_{it}, I_{it}; x^\star_t) + \delta_t \mathbb{E}_t[\mathcal{V}(x_{it} + I_{it} + \tilde{\varepsilon}_{it}; x^\star_{t+1})|\omega_t, w_{it}]. \quad (8)$$

where δ_t is the discount factor corresponding to interest rate r_t, i.e. $\delta_t = (1+r_t)^{-1}$. It is shown in the appendix that

Lemma 1 *Firm i's equilibrium investment at time t is given by*

$$I_{it} = \hat{x}_{it+1} - x_{it} - r_t. \quad (9)$$

The firms' technology levels at $t+1$ are normally distributed with mean

$$\mu_{t+1} = \mu_t + \frac{z_t - r_t + \tau\eta_t + \varepsilon^\star_t}{1-\lambda} \quad (10)$$

corresponding to the date-$t+1$ AGT level of the economy, and variance

$$\sigma^2_{t+1} = \sigma^2_\varepsilon + (1-\tau)^2 \sigma^2_\nu \equiv \sigma^2_x.$$

According to (9), firms' investment strategy is to adapt their production process to their estimate of the most efficient technology diminished by the price of capital r_t, which stirs firms to disinvest. For firms with a low technology level, this strategy corresponds to buying more environmentally friendly equipments. For the others, their investment is directed in the opposite direction: because they have technologies more environmentally efficient than required next period according to their estimate of $x^\star_{t+1}$, they can save on new equipment spending by buying less expensive 'brown' technologies. On average, this heterogeneity in investment policies should somehow cancel out, but while this is true for idiosyncratic noises, the public signal η_t distorts firms choices in the same direction to an extent that depends on its reliability

τ: the better the reliability, the larger the distortive effect. These distortions modify stochastically the dynamic of the AGT index as given by (10), which otherwise is positively affected by the environmental policy z_t but decreases with the cost of capital r_t. The coordination problem that affects firms' investment choices translates in the dynamic of the AGT index (10) through a 'magnifying' factor $(1-\lambda)^{-1}$: the larger λ, the larger the effects of the exogenous shock, of the public signal and of the net cost of capital $z_t - r_t$. As $\mathbb{E}[\mu_{t+1}] = \mu_t + (z_t - r_t)/(1-\lambda)$, absent governmental incentives ($z_t = 0$), the larger λ, the lower the AGT index. Indeed, the ideal technology mix in that case is close to the next period AGT index when λ is close to 1. As investment is costly, each firm anticipates that other firms investments will be low, which leads to an equilibrium that tends to the lower bound of the AGT index.

Each period, the industrial sector can be thought of as a 'cloud' of firms with a technology level that is drawn from a normal distribution centered on the AGT index μ_t with standard deviation σ_x. Aggregate production net of investment is given by

$$c_t = q_t - \int I_{it} di = q_t - \mu_{t+1} + \mu_t. \tag{11}$$

At market equilibrium, c_t corresponds to total consumption.[7]

The environment

Production generates pollution which harms the environment, but this detrimental effect can be reduced if firms improve their technology parameter. This mechanism is embodied in the following dynamic of the environmental quality (EQ index)

$$e_{t+1} = \theta e_t + \xi \mu_t - \varphi q_t + \hat{e} \tag{12}$$

where $\theta < 1$ and $\hat{e} \geq 0$. Here e_t is the EQ at time t, and ξ and φ are the environmental factors associated to AGT level and to the GDP level respectively. $\hat{e}$ is the per period maximum regeneration capacity of the environment, the actual regeneration level reached at period t being $\hat{e} - (1-\theta)e_{t-1}$, which depends (positively) on the environmental inertia rate θ. Without human

[7] Observe that the firm's cost $(x_{it} - x_t^\star)^2/2$ does not appear in (11). Indeed, this cost corresponds to supplementary expenses like tax payments, payroll outlays, external services, etc., that are revenues for the other economic agents.

interferences, we would have $\mu_t = q_t = 0$ and the EQ index would eventually reach the pristine level $e_N = \hat{e}/(1-\theta)$. We suppose that $\varphi q_S > \hat{e}$, implying that absent an environmental policy, the environment will collapse once production is sufficiently large. We define a sustainable economy as a stationary state where the average EQ index is above some threshold level $\bar{e} \in [0, e_N]$. More precisely, given the dynamic of GDP (1) and of the environment (12),

Definition 1 *[Sustainable Stationary State] The economy at date T has reached a sustainable stationary state (SSS) if for all $t \geq T$, (μ_t, e_t, q_t) is distributed around (μ_S, e_S, q_S) where $q_S = g_0/(1-g)$, $e_S = [\xi\mu_S - \varphi q_S + \hat{e}]/(1-\theta)$ and $\mu_S \in (\underline{\mu}, \mu_M]$ with $\mu_M < \varphi q_S/\xi$ and $\underline{\mu} = \mu_M - (1-\theta)(e_N - \bar{e})/\xi$.*

In this definition, it is assumed that the AGT index admits an upper bound $\mu_M < \varphi q_S/\xi$: this condition states that the economy cannot reach a SSS with an EQ index greater than e_N. The lower bound on the AGT index $\underline{\mu}$ corresponds to the environmental threshold level $\bar{e}$ which is a tipping point that should not be overpassed too frequently: if pollution is too large too often, the resilience of the environment is at stake, i.e. abrupt shifts in ecosystems may happen with dire and irreversible consequences for society. To avoid such a perilous situation, the AGT index should exceed $\underline{\mu}$.

Consumers

There is a representative consumer who maximizes her intertemporal utility arbitraging between consumption and savings every periods. Her saving and consumption plans solve

$$\max_{c,S} \left\{ \mathbb{E}_{t_0} \sum_{t=t_0}^{+\infty} \beta^{t-t_0} u(c_t, e_t) : c_t = R_t + r_{t-1}S_{t-1} - s_t, s_t = S_t - S_{t-1} \right\}$$

where c_t and R_t are her date-t consumption and revenue, S_{t-1} her savings from the previous period, $r_{t-1}S_{t-1}$ the corresponding date-t earnings, s_t the savings adjustment of period t, and β the psychological discount factor. The Bellman equation corresponding to the consumer's problem can be written as

$$v(S_{t-1}; e_t) = \max_{s_t} u(r_{t-1}S_{t-1} + R_t - s_t, e_t) + \beta \mathbb{E}_t v(S_{t-1} + s_t; e_{t+1})$$

where S_t and s_i are the state and the control variables respectively. The first-order equation is given by

$$\frac{\partial u(c_t, e_t)}{\partial c} = \beta \mathbb{E}_t \left[\frac{\partial v(S_t; e_{t+1})}{\partial S} \right] \tag{13}$$

and the envelop theorem gives

$$\frac{\partial v(S_{t-1}; e_t)}{\partial S} = r_{t-1} \frac{\partial u(c_t, e_t)}{\partial c} + \beta \mathbb{E}_t \left[\frac{\partial v(S_t; e_{t+1})}{\partial S} \right].$$

Replacing the last term using (13), we get

$$\frac{\partial v(S_{t-1}; e_t)}{\partial S} = (1 + r_{t-1}) \frac{\partial u(c_t, e_t)}{\partial c}.$$

Taking the expectation and replacing in (13) yields

$$\frac{\partial u(c_t, e_t)}{\partial c} = (1 + r_t) \beta \mathbb{E}_t \left[\frac{\partial u(\tilde{c}_{t+1}, e_{t+1})}{\partial c} \right] \tag{14}$$

where $1+r_t$ on the RHS is factorized out of the expected value since the date-t interest rate is a known parameter. Expectation is taken over all possible date-$t+1$ consumption/net-production levels that depend on the consumer's expectation about the firms' investment decisions at that date. Equation (14) corresponds to the supply function of capital, while (10) is the demand side coming from firms. At the date-t equilibrium on the capital market, the interest rates embodied in (14) and (10) are equal, allowing us to determine the global equilibrium of the economy at that date.

We assume that the representative consumer forms rational expectations over the future states of the economy and, whenever it is necessary to complete the analysis, that her preferences are CARA and that consumption and environmental quality can be subsumed in a 'global wealth index' denoted $y_t \equiv c_t + \rho e_t$. Here, ρ is the implicit price of the environment, assumed to be the same whatever the GDP of the economy.[8] Under these assumptions,

[8]The marginal rate of substitution between economic wealth and the environment is thus constant in that case. It is a reasonable approximation as long as $g < 1$ (i.e. wealth is bounded by q_S) and the environment has not incurred dramatic changes (e_t is above $\bar{e}$). Assuming steady growth ($g > 1$), as the environmental quality is bounded upward, this price should increase at the same pace as GDP along the balanced growth path: $\rho_t = g^t \rho_0$.

we have $u(c_t, e_t) = -e^{-\gamma y_t}$ where γ corresponds to the coefficient of absolute risk aversion. Assuming that the global wealth index y_{t+1} is normally distributed (which is effectively the case at equilibrium), we get from (14) using $\mathbb{E}[e^{-\gamma\tilde{y}}] = e^{-\gamma(\mathbb{E}[\tilde{y}]-\gamma\mathbb{V}[\tilde{y}]/2)}$, $\beta = e^{-\psi}$ where ψ is the intrinsic discount factor, and $1 + r_t \approx e^{r_t}$, a supply function of capital satisfying

$$r_t = \psi + \gamma(\mathbb{E}_t[\tilde{y}_{t+1}] - y_t) - \gamma^2\mathbb{V}_t[\tilde{y}_{t+1}]/2 \tag{15}$$

which exhibits the familiar effects that determine the rental price of capital: the intrinsic preference for an immediate consumption ψ, the economic trend of the global wealth index that also encourages immediate consumption if it is positive, and as revealed by the last term, a precautionary effect that operates in the opposite direction and corresponds to a risk premium due to the uncertainty affecting the economy. Under these assumptions, it is possible to derive the Rational Expectation Equilibrium (REE) of this economy at each period anticipating the environmental policy that will be implemented by the government $\{z_{t+h}\}_{h=1,2,\ldots}$. More specifically, it is shown in the appendix that

Lemma 2 *Under a REE with preferences over global wealth and CARA utility, the dynamic of the technological parameter is linear, satisfying*

$$\mu_{t+1} = a_1\mu_t + a_2e_t + a_3q_t + a_4 + a_5(\tau\eta_t + \varepsilon_t^\star) + Z_t \tag{16}$$

where $a_1 < 1$*and* $a_1 > 0$ *if* $2 + (1-\lambda)/\gamma > \theta + \rho\xi$, *given by*

$$a_1 = \frac{3 - \theta + (1-\lambda)/\gamma - \sqrt{[1-\theta+(1-\lambda)/\gamma]^2 + 4\rho\xi}}{2}, \tag{17}$$

The other coefficients are deduced from

$$\begin{aligned} a_2 &= \frac{(1-a_1)(1-\theta)}{\xi}, a_3 = \frac{\gamma[1-g+\varphi(\rho-a_2)]}{1-\lambda+\gamma(2-a_1-g)}, \\ a_4 &= \frac{\gamma[\hat{e}(a_2-\rho) - g_0(1-a_3)] - \psi + \gamma^2\sigma_y^2/2}{1-\lambda+\gamma(1-a_1)}, \\ a_5 &= 1/[1-\lambda+\gamma(2-a_1)], \end{aligned} \tag{18}$$

and we have

$$Z_t = a_5\sum_{i=0}^{+\infty}(\gamma a_5)^i z_{t+i}. \tag{19}$$

The date-t estimate of the date-$t+1$ wealth index is normally distributed with variance

$$\mathbb{V}_t[\tilde{y}_{t+1}] = \sigma_y^2 = (1-a_3)^2\sigma_\kappa^2 + a_5^2(\tau^2\sigma_\eta^2 + \sigma_\star^2) \tag{20}$$

for all t.

In addition to the state variables (μ_t, e_t, q_t) and the realizations of the technology shock $\varepsilon_t^\star$ and of the noise η_t affecting the public signal, the dynamic of the green index (16) entails a forward looking term Z_t given by (19) that accounts for the anticipated effects of the environmental policy. This dynamic is thus consistent only if the policy plan is known.[9] Because of the linearity of (44), the AGT index $\tilde{\mu}_{t+2}$ as estimated at date t is Gaussian (while the date-t realizations of the shocks are known, $\tilde{\mu}_{t+2}$ depends on the next period shocks $\tilde{\kappa}_{t+1}$, $\tilde{\varepsilon}_t^\star$ and $\tilde{\eta}_{t+1}$). From (11), $\tilde{c}_{t+1}$ is thus normally distributed, resulting in global wealth index $\tilde{y}_{t+1}$ which is also Gaussian with variance given by (20). Knowing z_{t+h}, $h = 1, 2, \ldots$, it is possible to infer statistically the state of the economy at horizon h from an initial state at date t by applying recursively (16) together with (1) and (12), thanks to the normal distribution of the random shocks.

Using (10) or (15) and (16), it is possible at the beginning of each period to specify the distributions of the equilibrium interest rate that will prevail at that date and of the next period wealth index. More precisely, we have

Lemma 3 *Under a REE with preferences over global wealth and CARA utility, before the current period shocks and signals are known, the current interest rate and the next period wealth index are normally distributed. Their variances are given by*

$$\sigma_r^2 = (1-\lambda)^2 a_3^2 \sigma_\kappa^2 + [1-(1-\lambda)a_5]^2(\tau^2\sigma_\eta^2 + \sigma_\star^2) \tag{21}$$

and

$$\sigma_{y_{+1}}^2 = \sigma_y^2 + [(1-a_3)g + (1-a_1)a_3 + (a_2-\rho)\varphi]^2\sigma_\kappa^2 + (1-a_1)^2 a_5^2(\tau^2\sigma_\eta^2 + \sigma_\star^2)$$

respectively.

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[9]As detailed in the following section, the optimal policy supposes a constant revision of the schedule $\{z_{t+h}\}_{h>0}$ at each date t to account for the shocks that have affected the economy. That the original policy plan is likely to evolve should be anticipated by the consumers. We suppose however that the consumers consider Z_t as a weighted sum of the policy parameters as announced by the regulator at date t rather than the weighted sum of random variables.

Observe that the variance of the interest rate is different from $\gamma^2\sigma_y^2$. This is due to the fact that the two bracketed terms in (15) are correlated random variables when the current shocks are unknown: the expectation of $\tilde{y}_{t+1}$ without knowing the realization of $\tilde{y}_t$ is a random variable.[10] Similarly, the variance of the next period wealth index comprises two terms, the first one, σ_y^2, corresponding to next period shocks and the second one for the shocks of the current period, this latter term accounting for the correlation between $\tilde{y}_{t+1}$ and $\tilde{y}_t$.

While absent a governmental policy the environmental quality would fall, the economy could admit a whole set of sustainable stationary states if the government implements a policy that allows the economy to stay in the vicinity of such states (assuming one is ever reached). From (15) and (10), the interest rate that prevails at a stationary state is $r_S = \psi - \gamma\sigma_y^2/2$ and the government should implements each period a policy $z_S = r_S$ to encourage firms to adopt on average the same green technology level each period. To be consistent with a REE equilibrium, the putative stationary state and such a policy must satisfy(16). It is shown in the appendix that:

Lemma 4 *Under a REE with preferences over global wealth and CARA utility, the economy stays in a SSS reached at date T if the government implements policy $z_t = \psi - \gamma\sigma_y^2/2$for all $t \geq T$.*

Among the many possible stationary equilibria that can be sustained, the one with the largest quality index, and thus the largest environmental level, is the best one. No stationary equilibrium is reachable absent an environmental policy. In the following section, we investigate the policy that allows the economy to follow an optimal path toward any targeted stationary state.

3 Environmental policy

We consider a benevolent social planner who decides on economic and environmental policies in order to maximize the social welfare, i.e. the discounted

[10]Indeed, denoting $\mathbb{E}_{t-1}[\tilde{y}_{t+1}|\tilde{y}_t] = m(\tilde{y}_t)$, it comes from (15) that

$$\tilde{r}_t - \mathbb{E}_{t-1}[\tilde{r}_t] = \gamma\{m(\tilde{y}_t) - \mathbb{E}_{t-1}[m(\tilde{y}_t)] - (\tilde{y}_t - \mathbb{E}_{t-1}[\tilde{y}_t])\},$$

which gives

$$\sigma_r^2 = \gamma^2(\sigma_y^2 + \sigma_{y_{+1}}^2) - 2\gamma cov(m(\tilde{y}_t), \tilde{y}_t).$$

sum of expected utility levels reached by the representative consumer. Our focus is on a policy that modifies the dynamics of the ideal technology index (4), and thus the dynamic of the AGT level (10), through setting z_t, for all $t \geq t_0$ where t_0 is the first period the policy is designed and implemented. The decisions made by the social planner change the economic environment of firms, but due to the shocks, the social planner cannot perfectly control the economic environment that will prevail in the future. Even at period t_0, the realizations of the economic shocks (and of the public and private signals) are known after the social planner has determined its policy. The optimal policy modifies the expected path of the economy through the dynamic of the average technology level corresponding to (10), given by $\mathbb{E}[\tilde{\mu}_{t+1}] = \mathbb{E}[\tilde{\mu}_t] + (z_t - \mathbb{E}[\tilde{r}_t])/(1-\lambda)$, where $\tilde{r}_t$ is a random variable even when $t = t_0$. The policy design problem is formally equivalent to determining for each date $t \geq t_0$the supplemental level of investment $\iota_t = (z_t - \mathbb{E}[\tilde{r}_t])/(1-\lambda)$ which can be alternatively interpreted as the (expected) effect of a net subsidy on the rental price of capital (devoted to green technologies) that should be given to investors in this economy to obtain the AGT dynamic given by

$$\mathbb{E}[\tilde{\mu}_{t+1}] = \mathbb{E}[\tilde{\mu}_t] + \iota_t. \tag{22}$$

The optimal policy solves the following Bellman equation

$$\mathcal{W}\left(\mu_t, e_t, q_t\right) = \max_{\iota_t} \mathbb{E}_t[u(\tilde{c}_t, e_t)] + \beta \mathbb{E}_t[\mathcal{W}\left(\tilde{\mu}_{t+1}, \tilde{e}_{t+1}, \tilde{q}_t\right)] : (1),\ (11),\ (12),(22)]$$

where μ_t, e_t and q_t are state variables and ι_t the control variable. The optimal policy is thus a policy plan that is revised each period to account for the current states of the economy. As explained below, we consider nevertheless two cases of environmental policy in the following: the commitment case in which the policy-maker does not revise the policy $\{z_t\}_{t \geq t_0}$ as determined date t_0, and the no-commitment case, in which the optimal policy is revised each period (and should be as such perfectly anticipated by the consumers at a REE).However, even in this latter case, the social planner must implement the date t policy measures at the beginning of that period, before the realization of the shocks and the signals, anticipating that the consumer chooses her intertemporal consumption levels depending on the realized interest rate. Therefore, the optimal policy is determined through the path of the expected optimal interest rate $\mathbb{E}[\tilde{r}^\star_t]$ where $\tilde{r}^\star_t \equiv \tilde{r}_t - \mathbb{E}[\tilde{r}_t] - (1-\lambda)\iota_t = \tilde{r}_t - z_t$ denotes the random variable corresponding to the optimal cost of capital given the random shocks and signals. It is shown in the appendix that:

Proposition 1 *The optimal expected production/environment state satisfies*

$$r_t^e - (1+\theta) r_{t+1}^e + r_t^e r_{t+1}^e - 1 + \theta = \xi \frac{\mathbb{E}\left[\partial u(\tilde{c}_{t+2}, \tilde{e}_{t+2})/\partial e\right]}{\mathbb{E}\left[\partial u(\tilde{c}_{t+2}, \tilde{e}_{t+2})/\partial c\right]}, \tag{23}$$

where

$$r_t^e \equiv \frac{\mathbb{E}\left[\tilde{r}_t^\star \partial u(\tilde{c}_{t+1}, \tilde{e}_{t+1})/\partial c\right]}{\mathbb{E}\left[\partial u(\tilde{c}_{t+1}, \tilde{e}_{t+1})/\partial c\right]} = \mathbb{E}[\tilde{r}_t^\star] + \frac{cov\left(\tilde{r}_t^\star, \partial u(\tilde{c}_{t+1}, \tilde{e}_{t+1})/\partial c\right)}{\mathbb{E}\left[\partial u(\tilde{c}_{t+1}, \tilde{e}_{t+1})/\partial c\right]}. \tag{24}$$

The optimal path of the economy is thus determined through the path of the "corrected"expected optimal interest rate r_t^e that solves (23). Compared to $\mathbb{E}[\tilde{r}_t^\star]$, it takes account of the (positive) correlation between the marginal utility of consumption and the interest rate as shown in (24). Because $\mathbb{E}[\tilde{r}_t^\star]$ accounts for the effects of green technologies on the environmental quality, it should be lower than the interest rate $\tilde{r}_t$ that prevails at the financial market equilibrium.

To completely specify the optimal expected path of the economy, we assume in the following a constant implicit price of the environmental quality (ρ), a CARA utility function and rational expectations. As we have $\partial u(c,e)/\partial e = \rho \partial u(c,e)/\partial c$, (23) becomes

$$r_t^e - (1+\theta) r_{t+1}^e + r_t^e r_{t+1}^e = \rho\xi + 1 - \theta \tag{25}$$

which prescribes the evolution of the interest rate from one period to the next along an optimal path. Observe that this dynamic does not depend on the state of the economy (none of the state variable q_t, e_t and μ_t is involved). From its initial value $r_{t_0}^e$, it may converge to a long run level that solves

$$(r_\sharp^e - \theta) r_\sharp^e = \rho\xi + 1 - \theta. \tag{26}$$

The left-hand side of (26) is a quadratic equation which is positive for either $r_\sharp^e \leq 0$ or $r_\sharp^e \geq \theta$ while the right-hand side is strictly positive. Hence $r_\sharp^e$ may take two values, one being positive (and greater than θ) and the other negative. Fig. 1 depicts the situation: the parabola correspond to the left-hand side of (26) which crosses the x-axis at 0 and θ. The horizontal line corresponds to the right-hand side of (26). It is assumed that $\rho\xi < 2\theta$. The negative root of (26) is equal to -1 if $\rho\xi = 2\theta$ which would be the case if the intercept of the horizontal line were located at $1+\theta$. Only the negative root is indicated since it is shown in the appendix that

Proposition 2 *Under a REE with preferences over global wealth and CARA utility, the path of the optimal average interest rate may only converge to the negative root of (26). Denoting by A the square root of the discriminant of (26), i.e.*

$$A = \sqrt{(2-\theta)^2 + 4\rho\xi},$$

this root is given by

$$r^e_\sharp = -(A-\theta)/2 \tag{27}$$

and the solution of (25) is given by

$$r^e_t = r^e_\sharp + \frac{A(r^e_{t_0} - r^e_\sharp)k^{t-t_0}}{A - (1-k^{t-t_0})(r^e_{t_0} - r^e_\sharp)}, \tag{28}$$

for all $t \geq t_0$, where

$$k = \frac{2+\theta-A}{2+\theta+A}, \tag{29}$$

provided $r^e_{t_0} - r^e_\sharp < A$. Moreover, if $2\theta > \rho\xi$ we have $0 < k < 1$, $0 > r^e_\sharp > -1$, and $r^e_t > r^e_\sharp$ for all $t \geq t_0$ if $r^e_{t_0} - r^e_\sharp < A$. If $2\theta < \rho\xi$, we have $r^e_\sharp < -1$ and $0 > k > -1$. The corresponding optimal expected interest rate is given by

$$\mathbb{E}_t[\tilde{r}^\star_t] = r^e_t - (1-\lambda)a_3\gamma\left(\sigma^2_{y_{+1}} - \sigma^2_y\right) - \gamma(1-a_1)^2 a_5^2(\tau^2\sigma^2_\eta + \sigma^2_\star) \tag{30}$$

Because it accounts for the beneficial effects of investing in green technology, the long run optimal cost of capital is negative while the financial interest rate is most certainly positive.[11] In particular, when the environmental quality inertia θ is low, i.e.$2\theta < \rho\xi$, this cost of capital is lower than -1 as explained above. This situation seems however quite extreme: firms would be more than compensated for the losses they incur in investing in green technology. Moreover, the expected interest rate would oscillate around its long term value along the optimal path creating fluctuations that are at odds with the consumer' desire to smooth her consumption path. In the following discussion, our focus will be on the more probable case of a monotonic convergence, i.e. $2\theta > \rho\xi$. The term (28) is the best estimate that the social planner can make today (at time t_0) on the expected path of the optimal

[11] Note that at steady state, the interest rate given by (15) could be negative if $\psi < \gamma\sigma^2_{\tilde{y}_S}/2$, i.e. if the precautionary effect due to economic shocks is larger that the intrinsic preference rate. This is less probable along a path of the economy with a positive economic trend due to the wealth effect.

interest rate given an initial value $r^e_{t_0}$ deduced from the current state of the economy. It allows him to forecast the desirable states that the economy must reach (in expectation) in the following periods, and thus to decide on the policy measure z_t to be implemented. Observe that whatever the initial value of the expected interest rate, it converges to the same value given by (27) which depends on the implicit price of the environment ρ and on the parameters that govern the dynamic of the environment: the environmental inertia θ and the parameter that captures the impact of the technology index on the environment ξ. However, this convergence necessitates that the initial expected interest rate gap $r^\star_{t_0} - r^\star_\sharp$, where $r^\star_\sharp = r^e_\sharp - \gamma^2(\sigma^2_{y+1} - \sigma^2_y)$, is not too large. It should be stressed that Prop. 2 only described the convergence of the optimal interest rate to a long-run level $r^\star_\sharp$ which is different from the SSS level $r_S = \psi - \gamma\sigma^2_y/2$. In particular, as long as the government implements $r^\star_\sharp$, the expected AGT index increases (if it has not reached its maximum value μ_M).

To characterize the policy, it is convenient to use the normalized gap between the expected interest rate at horizon h and its long run level which is defined as

$$d_{t_0+h} \equiv (r^e_{t_0+h} - r^e_\sharp)/A = (\mathbb{E}[r^\star_{t_0+h}] - r^\star_\sharp)/A \tag{31}$$

and belongs to $[0, 1)$ if $d_{t_0} < 1$ as stated in Proposition 2. It is shown in the appendix that

Proposition 3 *Under a REE with preferences over global wealth and CARA utility, the normalized gap at horizon h from t_0 can derived recursively using $d_{t_0+h} = f(d_{t_0+h-1})$ where*

$$f(x) \equiv kx/[1 - (1 - k)x] \tag{32}$$

with k as defined in Prop. 2. This gap converges to 0 *provided* $d_{t_0} < 1$.

A phase diagram of this recursion is given fig. 2 which depicts the relationship $d_{t+1} = f(d_t)$ in the (d_t, d_{t+1}) plane and where the bisector is used as a "mirror"to project the value d_{t+1} back on the d_t axis. Function $f(\cdot)$ is convex, with a vertical asymptote for $d_t = 1/(1 - k)$ but relevant values for d_t are lower than 1. We have $f(0) = 0$ and $f(1) = 1$ with $f'(0) = k < 1$ and $f'(1) = 1/k > 1$, hence the bisector is located above function $f(\cdot)$ for d_t belonging to $[0, 1]$ and below it for negative values of d_t. From an initial gap value d_{t_0} (either negative or positive as depicted), the arrows allow to follow the recursion toward 0. One can see that convergence is fast for large negative values of d_{t_0} due to the fact that $\lim_{x\to-\infty} f(x) = -k/(1-k)$, hence the horizontal asymptote below the curve of $f(\cdot)$.

Hence, applying recursively $d_{t_0+h} = f(d_{t_0+h-1})$ gives the expected value of the optimal interest rate from one period to the next. As discussed in the following, (32) can also be used as a tool to update the environmental policy each period t taking as initial value the observed current gap d_t.

4 Policy implementation and Environmental forecast

Assuming that the social planer implements the policy characterized above, the expected paths of the AGT and EQ indexes can be anticipated using Prop. 3. To initiate the recursion, we suppose that the government is able to determine the expected value of the interest rate at date $t_0 - 1$, the period before the policy is implemented, which allows it to derive the initial normalized gap. It corresponds to the normalized difference between the expected interest rate r_{t_0-1} and $r_\sharp^\star$: $d_{t_0-1} = (r_{t_0-1} - r_\sharp^\star)/A$(that we assume lower than 1). The normalized gap of the first period is deduced from this level using $d_{t_0} = f(d_{t_0-1})$. It allows the social planner to determine the first period expected optimal interest rate using $\mathbb{E}_{t_0}[\tilde{r}_{t_0}^\star] = r_\sharp^\star + Ad_{t_0}$, and the sequence of all future ones applying $\mathbb{E}_{t_0}[\tilde{r}_t^\star] = r_\sharp^\star + Ad_t$ with $d_t = f(d_{t-1})$ for all $t > t_0$. The corresponding expected optimal paths of the AGT and EQ indexes are deduced from (10) and from (12) taken on expectation and solved recursively. Due to the shocks affecting the economy, these levels correspond to the expectations of Gaussian random variables. To characterize their distributions, it is possible to use the fact that the normalized gap at horizon h corresponds to the expectation of the Gaussian random variable

$$\tilde{d}_{t_0+h} = d_{t_0+h} + \tilde{v}_{t_0+h}$$

where $\tilde{v}_{t_0+h} = (\tilde{r}_{t_0+h} - \mathbb{E}_{t_0+h}[\tilde{r}_{t_0+h}])/A$. From Lemma 3, we know that these shocks are identically and independently distributed according to $\tilde{v}_{t_0+h} \sim \mathcal{N}(0, \sigma_r^2/A^2)$ for all h. It is shown in the appendix that

Proposition 4 *The AGT index $\tilde{\mu}_{t+h}$ under the environmental policy follows* *a Gaussian random walk with mean*

$$\mathbb{E}_{t_0}[\tilde{\mu}_{t_0+h}] = \mu_{t_0} - \frac{hr_\sharp^\star + \sum_{i=0}^h f^i(d_{t_0})}{1-\lambda} \tag{33}$$

and variance $\mathbb{V}_{t_0}[\tilde{\mu}_{t_0+h}] = h\mathbb{V}[\tilde{\mu}_{t+1}]$ *for all* $1 \leq h < T$*, where* $T = \min\{h : \mathbb{E}_{t_0}[\mu_{t+h}] \geq \mu_M\}$ *corresponds to the expected number of periods necessary to reach the maximum AGT value* μ_M*. It is bounded above: we have*

$$T < \frac{1}{-r_\sharp^\star}\left\{(1-\lambda)(\mu_M - \mu_{t_0}) + \frac{d_{t_0}}{1-d_{t_0}}\frac{1}{1-k}\right\}.$$

For all period $h \geq T$*,* $\mathbb{E}_{t_0}[\mu_{t_0+h}] = \mu_M$ *provided that* $z_t = \mathbb{E}_{t_0}[\tilde{r}_t]$ *for all* $t \geq T$*. The corresponding EQ index* $\tilde{e}_{t_0+h}$ *follows a Gaussian random walk with mean satisfying*

$$\begin{aligned} e_S - \mathbb{E}[\tilde{e}_{t_0+h}] &= \theta^h(e_S - e_{t_0}) + \xi\left(\mu_S - \mu_{t_0}\right)\frac{1-\theta^h}{1-\theta} - \frac{\xi|r_\sharp^\star|}{1-\lambda}\frac{h(1-\theta)-1+\theta^h}{(1-\theta)^2} \\ &\quad + \frac{A\xi}{1-\lambda}\sum_{i=1}^{h-1}\theta^i\sum_{j=1}^{h-1-i} d_{t+j} - \varphi(q_S - q_{t_0})\frac{g^h-\theta^h}{g-\theta} \end{aligned} \tag{34}$$

and variance given by

$$\mathbb{V}[\tilde{e}_{t+h}] = a_5^2\xi^2(\tau^2\sigma_\eta^2 + \sigma_\varepsilon^2)\sum_{i=0}^{h-1}\left(\frac{1-\theta^i}{1-\theta}\right)^2 + \sigma_\kappa^2\sum_{i=0}^{h-1}\left(-\varphi\theta^i + a_3\xi\frac{1-\theta^i}{1-\theta}\right)^2$$

for all $h < T$*. Finally, the policy schedule as anticipated at the beginning of the govermental intervention* $\{z_t\}_{t\geq t_0}$ *is derived from*

$$\begin{aligned} z_{t_0+h} &= \psi - \gamma^2\sigma_{y+1}^2 - r_\sharp^\star + \frac{A}{1-\lambda}\left(\gamma d_{t+1} - (\gamma+1-\lambda)\,d_t\right) \\ &\quad + \gamma[\rho(\theta-1)e_t + \xi\rho\mu_t + (g-\varphi\rho-1)gq_{t-1} + (g-\rho\varphi)g_0 + \rho\hat{e}] \end{aligned} \tag{35}$$

The maximum level of the AGT index is thus reached after a finite number of periods. This is true in expectation, but of course the actual number of periods necessary may be lower or larger than the estimated level at date t_0 because the technology level follows a Gaussian random walk with a variance at horizon h being h times the variance from one period to the other. The expected time horizon for the EQ level to reach its stationary state is also different from T since it depends on its initial gap and of the one of the GDP too. However, given the characteristics of the random walks followed that the AGT and the EQ indexes, it is possible to assign confidence intervals to the levels they may reach at horizon h. Reciprocally, for a given environmental target level, one can deduce the probability that it is reached at horizon h.

Furthermore, form the second derivative of the environmental gap (34) with respect to the time horizon h, we get that the environmental gap is concave if

$$(e_S - e_{t_0}) - \frac{\xi(\mu_S - \mu_{t_0})}{1-\theta} - \frac{\xi|r_\sharp^\star|}{(1-\lambda)(1-\theta)^2} + \frac{\varphi(q_S - q_{t_0})}{g-\theta} < 0. \tag{36}$$

Hence, under this condition on the parameters of the economy and on its initial state (at time t_0), the environmental quality keeps deteriorating for some periods and reaches a minimum before starting raising again toward the stationary state e_S.

5 Policy commitment and sustainability

The definition of the environmental policy presents a "commitment versus discretion"dilemma that resembles the one affecting the design of inflation reduction policies which is discussed at length in the monetary economics literature.[12] Indeed, we have supposed that the social planer announces and commits to a policy that is defined once and for all at date t_0, neglecting the extent of the shocks that affect the economy as the future comes to pass. However, such a commitment suffers a credibility problem: in case of a large negative shock, say at date t_1, increasing the stringency of environmental regulations as planned date t_0 can only worsen the current fate of society. In such a situation, the social planer may thus be willing to re-consider his initial plans and formulate new environmental targets for the foreseeable future (and the more distant one), still maximizing the social welfare and thus using (28)

[12]This dilemma was first pointed-out by Kydland & Prescott (1977). Compared to the monetary economics literature focusing on inflation stabilization policies (see Clarida et al., 1999, for a review), our problem is to determine the paths of state variables (particularly μ_t and e_t) toward their steady state levels, which is different from reducing fluctuations around a stationary state. Moreover, the analysis of inflation targeting often reduces the policy-maker's objective to minimizing a loss function rather than maximizing the social welfare. See Woodford & Walsh (2005) for an extensive exposition of the microfoundations of the loss function approach.

to define these levels, but with initial interest rate gap the one effectively reached at date t_1, $r^\star_{t_1} - r^e_\sharp$, rather than the one predicted for this period at date t_0. The argument is also valid in the case of a positive shock: the social planer can take advantage of the good economic conditions to tighten a bit more the environmental policy. This should not be too detrimental in the current period and could increase the future well-being' if this policy updating is operated using (28). In fact, as long as such "discretionary"updates are obtained through the policy rule (28), it seems even socially desirable to "fine tune"the environmental policy every period.[13] Another reason makes this constant updating process more appealing: only the expected optimal cost of green technology is determined by (28), and it is highly unlikely that the realized interest rate at any date corresponds to the expected value estimated before. Because of the discrepancy between the realized value of the interest rate and its expected value, firms' investment in green technology differs from the estimate, and so does the next period expected interest rate. While it seems that these policy revisions should somehow cancel out, good shocks compensating for bad ones so that the path of the economy stays in the vicinity of the one initially predicted, we show in this section that the resulting effect of such an updating process is not neutral: it increases the expected interest rate gap on average, and thereby delays the convergence of the economic toward its desirable steady state. We detail this effect in the following considering the case $k > 0$, hence $\rho\xi < 2\theta$.

At the beginning of period $t+1$, the social planer knows the realization v_t of $\tilde{v}_t$. He can re-evaluate the policy using as a revised gap value $d^r_t = d_t + v_t$ if $v_t < 1 - d_t$ and d^r_t arbitrarily close to but lower than 1 if $v_t \geq 1 - d_t$. This revision of the policy leads to the updated expected path

$$d_{t+h|t} = f^{h-1}(d^r_t)$$

which differs from d_{t+h}. Such an updating procedure could be done each period and allows the government to adapt the environmental policy to the effective path of the economy, which may be very different from the one envisioned at date t_0. Besides, changes in the policy modify the path of

[13] There are also "technical" arguments: in the parlance of game theory, commitment is not subgame perfect. It could be justified in a setup where changes in the environmental policy influence the economic agents' beliefs, and thus their behaviors. By sticking or not to his plans, the decision-maker builds a reputation that, in fine, affects the economic outcome. This is not accounted for in a context restricted to Markov perfect equilibria.

the economy, resulting in non trivial deviations compared to the original estimate.

It is possible to obtain a rough assessment of the effect of this updating process using[14]

$$f(d_t + v_t) \approx f(d_t) + f(v_t) = d_{t+1} + f(v_t).$$

At date $t+2$, compared to the initial estimate, we get

$$d_{t+2|t+1} = f(d_{t+1|t} + v_{t+1}) \approx f(d_{t+1}) + f(v_{t+1} + f(v_t)) \approx d_{t+2} + f(v_{t+1}) + f^2(v_t)$$

and recursively

$$d_{t+h|t+h-1} \approx d_{t+h} + \sum_{i=0}^{h-1} f^i(v_{t+h-i}).$$

We thus obtain

$$\mathbb{E}_{t+h-1}[d_{t+h}] \approx d_{t+h} + \mathbb{E} \sum_{i=0}^{h-1} f^i(v_{t+h-i}),$$

and, using $f^i(x) = k^i x \sum_{j=0}^{+\infty} [(1-k^i)x]^j$,

$$\mathbb{E}_{t+h-1}[d_{t+h}] \approx d_{t+h} + \sum_{i=0}^{h-1} k^i \sum_{j=0}^{+\infty} (1-k^i)^j \mathbb{E}[\tilde{v}^{j+1}].$$

Because odds moments of the centered normal distribution are null while even ones are positive (i.e. $\mathbb{E}\tilde{v}^{j+1} = 0$ for $j = 2, 4, 6, \ldots$, and $\mathbb{E}\tilde{v}^{j+1} > 0$ for $j = 1, 3, \ldots$), it comes $\mathbb{E}_{t+h-1}[d_{t+h}] > d_{t+h}$. Using a second-order approximation (a sum up to $j = 2$) yields

$$\begin{aligned} \mathbb{E}_{t+h-1}[d_{t+h}] &\approx d_{t+h} + \sum_{i=0}^{h-1} k^i (1-k^i) \frac{\sigma_r^2}{A^2} = d_{t+h} + \frac{\sigma_r^2}{A^2} \left[\frac{1-k^h}{1-k} - \frac{1-k^{2h}}{1-k^2} \right] \\ &= d_{t+h} + \frac{\sigma_r^2}{A^2} \frac{k(1-k^{h-1})(1-k^h)}{1-k^2}. \end{aligned}$$

[14] We have (detail in the appendix)

$$f(d_t) + f(v_t) - f(d_t + v_t) = -\frac{(1-k) d_t v_t [f(d_t + v_t)(1-k) + 2k]}{1 - (1-k)(d_t + v_t) + (1-k)^2 d_t v_t}$$

where $|k| < 1$, $|d_t| \leq 1$ and relevant values for v_t verifying $0 \leq v_t + d_t \leq 1$. The approximation is better for k close to 1 (and trivially so for k close to 0).

For large time horizon, we get

$$\lim_{h\to+\infty} \mathbb{E}_{t+h-1}[d_{t+h}] \approx \lim_{h\to+\infty} d_{t+h} + \frac{\sigma_r^2}{A^2}\frac{k}{1-k^2}.$$

Notice that these approximations are made assuming that v_t could be symmetrically positive or negative while the revision process is asymmetric. This result suggests that “upward“ revisions are larger than “downward“ ones. This is illustrated Fig 2: due to the convexity of f, large positive values of d_0 initiate a path toward 0 that is longer than for negative ones.

6 Numerical simulations

In the previous sections, we obtained close form expressions of the optimal policy and of the dynamics of the main economic and environmental variables of the model that we use in the following simulations to assess the relative impact of the different parameters.

6.1 Calibration

In the following, the EQ index e_t is defined as the difference between a “tipping point” level of CO_2 in the atmosphere ℓ_M (i.e. a threshold above which the dynamic of the climate is irreversibly changed and no return to the pre-industrial state of the atmosphere is possible even if green technologies allows us to completely abate GHG emissions) and the level of GHG at date t, ℓ_t, expressed in CO_2 equivalent: $e_t = \ell_M - \ell_t$. [15] Hence, e_t can be thought of as a global “carbon budget” at date t relative to a global disaster level. Parameter θ is determined from the IPCC Fifth Assessment Report which estimates that 100 years after a 100 Gt CO_2 pulse in the preindustrial atmosphere, there would remain 40% of the 100 Gt CO_2 emitted, while after 1000 years there would remain 25%. Accordingly, denoting by $\hat{\ell}$ the preindustrial level of CO_2, after an initial period $\ell_0 = \hat{\ell} + 100$, we have $\ell_{100} = \hat{\ell} + 40$ and

[15] The unit used in the following is the gigatonne or Gt shorthand, i.e. 10^9 metric tons. One may also express theses levels by atmospheric concentration, the unit being the part per million or ppm shorthand, i.e. 0.01%. Each ppm represents approximately 2.13 Gt of carbon in the atmosphere as a whole. Conversion values can be found on the dedicated US department of energy website http://cdiac.ornl.gov/pns/convert.html#3.

$\ell_{1000} = \hat{\ell} + 25$. Using (12) without industrial interferences and solving the recursion gives

$$\ell_t = \theta^t(\hat{\ell} + 100) + (1 - \theta^t)\ell_M - \frac{1 - \theta^t}{1 - \theta}\hat{e}.$$

Using the IPCC's estimations for $t = 100$ and $t = 1000$ we obtain

$$\frac{40 - 100\theta^{100}}{1 - \theta^{100}} = \frac{25 - 100\theta^{1000}}{1 - \theta^{1000}}$$

independently of the choice of $\hat{\ell}$ and ℓ_M, which can be expressed as $1 - 5x + 4x^{10} = 0$ where $x = \theta^{100}$. We get $\theta \approx (1/5)^{1/100} \approx 0.984$. [16] Parameter $\hat{e}$ is deduced assuming that the long term equilibrium $e_N = \ell_M - \hat{\ell}$ was reached before industrialization using $\hat{e} = (1 - \theta)e_N$. Considering that $\hat{\ell} = 596.4$ Gt CO_2 (280 ppm) and $\ell_M = 1491$ Gt CO_2 (700 ppm), we obtain $\hat{e} \approx 6.72$.[17]

Since our goal is to model the greening of the production process through clean technology adoption, we define φ as the effective carbon intensity of our reference year, and see how the adoption of clean technology over time allows to alleviate the carbon footprint of firms. Accordingly, the initial AGT index μ_t is set to zero at the reference year and ξ is set to φ. Following Acemoglu et al. (2012), we use World bank data about world emissions and GDP to compute φ. Taking 2005 as the reference year, we get (GDP is expressed in billions – 10^{12}– of 2015 US$)

$$\varphi = \frac{1}{2}\frac{\text{World emissions}_{2005}}{\text{World GDP}_{2005}} = \frac{1}{2}\left(\frac{29,7 \text{ Gt}}{\$47\text{B}}\right) = 0.316 \text{ Gt/\$B},$$

where only half of CO_2 emissions is assumed to affect the CO_2 atmospheric concentration.

Parameters of the GDP dynamics (1) are set to $g = 0.99$ and $g_0 = 0.2$. These values correspond to a growth rate of 3.1% for the first year (2005)

[16] An obvious root of this equation is $x = 1$. The other roots are complex.

[17] According to the IPCC Fifth Assessment Report, 700 ppm lead to a temperature increase of approximately 4°C, a situation where "many global risks are high to very high." However, several tipping points are considered by climatologists. Candidates include levels corresponding to an irreversible melt of the Greenland ice sheet, the dieback of the Amazon rainforest and the shift of the West African monsoon. Acemoglu et al. (2012) use the atmospheric CO_2 concentration that would lead to an approximate 6°C increase in temperatures. Stern (2007) reports that increases in temperature of more than 5°C will lead among other things to the melting of the Greenland Ice Sheet.

and a growth rate of 0.9% in year 50 and then 0.4% in year 100. Without any pollution abatement, this calibration of the production and the environmental dynamics leads the environment to collapse (e_t reaches zero) within 54 years (in expectation). As a comparison, assuming a constant growth rate of 2% would lead the economy to collapse within 52 years. Therefore given the scale of our calibration, the finiteness of the economy should not have a major impact on the environmental dynamics: the growth rate of the economy decreases in a reasonable scale of time compared to our environmental dynamics.[18]

Risk aversion coefficient γ is set to 3 and the marginal rate of substitution between environment and consumption goods $\rho < \theta(1+\lambda)$. Following the debate on the discount rate between Stern and Nordhaus, we use two different values: $\beta = 0.999$ (Stern) and $\beta = 0.985$ (Nordhaus).

Finally we make the values for the variances of the various shocks σ_κ^2, σ_ν^2, σ_η^2 vary between 0.2 and 3. In particular, comparative static exercise on τ allows us to detail the impact of varying the relative precisions of the public signal versus the private one, even though the realization of the private signals do not enter the equilibrium dynamics of the model.

Results

Using our baseline calibration, Fig. 3 and Fig. 4 illustrate the dynamics of the EQ and the AGT indexes respectively, starting from the first period of the implementation of the environmental policy. In these figures, the expected paths and the upper and lower limits of a 95% confidence interval around these paths are provided. Stochastically generated shocks allow us to illustrate the difference between a laissez-faire situation and the corresponding "realized" paths of the indexes under the environmental policy.

Fig. 5 illustrates the per period investment level under laissez-faire and under the environmental policy. As it is easily noticed, investment is negative under laissez-faire, that is, firms tend to invest increasingly in polluting technologies, while the level of investment in green technology quickly converges to the optimal one (the opposite of the long run interest rate $r_\sharp^\star$) under the policy. Fig. 6 shows total welfare dynamics, still comparing laissez-faire

[18] In Acemoglu et al. (2012), the innovation function is calibrated so as to obtain a 2% long run growth. In DICE 2010, the economy grows at a rate equal to 1,9% from 2010 to 2100 and 0,9% from 2100 to 2200.

to the policy. Finally, Fig. 7 show the stochastic dynamics of the interest rate, in comparison with the targeted (expected) interest rate and the corresponding policy path $\{z_t\}_{t=t_0}^T$.

There are two aspects in the welfare impact of an environmental policy that can be analyzed here. First, implementing a subsidy for AGT has the immediate and most direct effect of increasing the rate of investment in green technologies, thus introducing a trade-off between current consumption and future environmental quality. Since our model disentangles green technology investment from productivity growth, there is no stock effect of increasing investment on intertemporal consumption. On the reverse, the environmental quality is a stock and reducing the size of the negative impact of production on the environment has long term effects on welfare through environmental quality accumulation. This prevents us from measuring in a very precise way the welfare impact of the policy in terms of intertemporal trade-off, but allows us to analyze the impact of our key parameters on the optimal rate of investment on AGT, which represents the short term cost of implementing the policy, and thus its main social and political impediment.

Fig. 8a to 8f illustrate the impact of the policy implementation on consumption (and symmetrically on investment levels). A higher consumption differential between the laissez-faire and the policy situations means that the policy implies a higher investment level and thus a higher cost in terms of postponed welfare. Fig. 8a is the baseline case, while Fig. 8b to 8d show the effect of changing the extent of technological "stickiness" λ. We observe that the higher λ, the lower the optimal consumption, that is, when coordination plays a major role in determining technologies profitability (and thus when adoption is very volatile), the optimal investment level is higher. Its sudden drop around the 100th period after t_0 in Fig. 8d shows that the economy has reached its stationary state much quicker than in the other scenarios. This illustrates the tradeoff between short term consumption decrease and long run environmental benefit when evaluating the welfare cost of a policy.

Fig. 8e shows the impact of postponing the implementation of the policy (here the policy starts after 20 periods instead of 5): optimal investment is lower.

Fig. 8f shows the impact of the magnitude of shocks that hit the economy: with small shocks' variances, optimal investment is slightly higher than with higher ones.

This first effect on short term welfare can be completed with the more innovative analysis of the threat that the possible occurrence of large environmental damages puts on welfare. Our modeling of uncertainty allows us to derive the expected trends of our main variables and the confidence intervals in which they are lying. Of particular interest is the convexity of the environmental quality path overtime: once the policy is implemented, the EQ index decreases for a few years, then reaches a minimum before turning up toward a recovery. As noted in (36), the condition for such a convex pattern depends on initial conditions and on the values of some parameters of the model. Numerical simulations show that a convex pattern is observed under a wide range of parameter values and scenarios.

This is of particular interest in our welfare analysis because while we assume that there exists a tipping point that environmental quality shouldn't cross in order to prevent a disaster, neither individual agents nor the social planner encompass such a risk in their behaviors. The linear environmental constraint considered by the social planner corresponds to the dynamics of a regular environment, away from extreme regions in which the dynamics may be highly non linear and subject to large and highly unpredictable shocks. Thus, the "optimal" policy implemented by the social planner contributes to solving the public good provision problem only when the environmental quality remains located between reasonable bounds that depend on the parameter of the environmental dynamics. It does not account however, for the probability of occurrence of an environmental disaster.

Derivation of the social carbon price

The marginal rate of substitution between environmental quality and consumption, ρ, may be thought of as some private value corresponding to the private disutility that agents encounter from climate change (or any other kind of pollution effects).[19] However, it is likely to be underestimated compared to the actual marginal contribution of environmental quality to global welfare. As a measure of the social marginal rate of substitution between environmental quality and consumption, ρ should encompass all the services provided by a good environmental quality and in particular, the fact that there exists tipping points beyond which the welfare consequences of environmental degradation are dire and irreversible. Indeed, whatever the value of ρ, there is always a non-zero probability that the actual environmental

[19]Disutility may come from a direct impact on welfare, due for example to frequent extreme weather events, or a more indirect one, due to private awareness and concern for environmental issues.

path hits a critical threshold under the optimal policy. Of particular interest is thus the value of ρ that ensures that an environmental disaster is avoided with at least a certain level of confidence that we denote ρ_s in the following.[20] Fig 9 shows the impact of ρ_s on the probability of hitting a tipping point corresponding to a CO2 concentration of 700 ppm (leading to an increase of atmospheric temperature of approximately 4°C) or 450 ppm (leading to an increase of atmospheric temperature of approximately 2°C). As we would expect, the higher the marginal rate of transformation between consumption and environmental quality, the lower the probability of hitting a tipping point (the higher the probability of avoiding it).

We can compare these values to the social costs of carbon found in the literature. Nordhaus (2007), finds a SCC of 11,5 \$/tCO2 for 2015 while more recently, with similar calibration for the rate of pure time preference, Traeger (2015) and Golosov et al. (2014), find SCC of respectively 15,5 and 16,4 \$/tCO2. Another interesting parameter in our calibration is the measure of technological spillovers λ.

Fig 10 shows how λ affects the relationships between ρ_s and the probability of hitting the 450 ppm tipping point (the tendencies are similar for a different tipping point). The higher λ, the lower the probability of hitting the tipping point.

Fig 11 shows how λ affects the value of ρ_s necessary to avoid hitting a tipping point with at least 98% chance. Again quite intuitively, the higher λ, the lower ρ_s needs to be to avoid hitting the tipping point with sufficient confidence.

7 Conclusion

In this paper, we analyze the process of green technology adoption when network complementarities are at work and when the economy is subject to several sources of uncertainty. We find that both fundamental uncertainty and coordination motives for investing firms tend to increase the volatility

[20] In all the following results, the private value of environmental quality ρ has very little impact and therefore we keep it constant, equal to 1 \$/tCO2, which is rather small compared to ρ_s. The reason why it does not affect much the results is relatively straightforward: agents do not internalize the effect of their investment on future environmental quality and thus on future welfare. Therefore, even though environmental quality impacts private welfare, it has very little impact on individuals' behavior.

of the AGT path and thus of the other main variables of the economy (environmental quality, consumption, welfare). We assume that a social planner implements a subsidy to AGT in order to force agents to internalize the welfare impact of using polluting technologies and harming the environment. The optimal policy includes a precautionary motive because the social planner does not observe future shocks and thus sets the subsidy higher than it would be needed without uncertainty. Such a precautionary motive is higher when the economy is more volatile, and therefore, high uncertainty or high complementarities in adoption make the optimal green technology investment (and consequently, postponed consumption) higher. However, when we assess the probability of occurrence of an unanticipated environmental disaster along the transitory path toward a sustainable economy, we find instead that higher complementarities decrease the risk of hitting an environmental disaster, which has a positive impact on welfare.

We finally briefly discuss the no commitment case in which the policy may be constantly re-evaluated to past observed shocks, and find that due to the convexity of the policy adjustment function, policy readjustments would allow to smooth the transition path toward a sustainable steady state, and make converge slower.

Possible extensions of this work would be to endogenize the productivity growth process in order to better assess the intertemporal trade-off between AGT and productivity improving investment, as well as between current consumption and future environmental quality. Further extensions would be to study global coordination problems between trading countries with global environmental issues and imperfect technology spillovers.

References

Acemoglu, D., Aghion, P., Bursztyn, L., & Hemous, D. (2012). The environment and directed technical change. *American Economic Review*, 102(1), 131–66. 1, 6.1, 17, 18

Angeletos, G.-M. & Werning, I. (2006). Crises and prices, information aggregation, multiplicity, and volatility. *American Economic Review*, 96(5), 1720–1736. 3

Bovenberg, A. L. & Smulders, S. (1995). Environmental quality and pollution-augmenting technological change in a two-sector endogenous growth model. *Journal of Public Economics*, 57, 369–391. 1

Bretschger, L. & Vinogradova, A. (2014). *Growth and Mitigation Policies with Uncertain Climate Damage*. Working Paper 5085, CESifo. 1

Clarida, R., Galí, J., & Gertler, M. (1999). The science of monetary policy: A new keynesian perspective. *Journal of Economic Literature*, 37, 1661–1707. 12

De Zeeuw, A. & Zemel, A. (2012). Regime shifts and uncertainty in pollution control. *Journal of Economic Dynamics and Control*, 36, 939–950. 1

Eaton, J. & Kortum, S. (1999). International technology diffusion: Theory and measurement. *International Economic Review*, 40(3), 537–570. 1

Fischer, C. & Heutel, G. (2013). Environmental macroeconomics: Environmental policy, business cycles, and directed technical change. *Annu. Rev. Resour. Econ.*, 5(1), 197–210. 1

Gerlagh, R. & Kuik, O. (2007). *Carbon Leakage with International Technology Spillovers*. Working Paper 2007.33, Fondazione Eni Enrico Mattei. 1

Golosov, M., Hassler, J., Krusell, P., & Tsyvinski, A. (2014). Optimal taxes on fossil fuel in general equilibrium. *Econometrica*, 82(1), 41–88. 1, 6.1

Heutel, G. (2012). How should environmental policy respond to business cycles? optimal policy under persistent productivity shocks. *Review of Economic Dynamics*, 15, 244–264. 1

Jensen, S. & Traeger, C. P. (2014). Optimal climate change mitigation under long-term growth uncertainty: Stochastic integrated assessment and analytic findings. *European Economic Review*, 69, 104–125. 1

Kydland, F. E. & Prescott, E. C. (1977). Rules rather than discretion: The inconsistency of optimal plans. *The Journal of Political Economy*, (pp. 473–491). 12

Lucas, R. E. (2009). Ideas and growth. *Economica*, 76(301), 1–19. 1

Lucas, R. E. & Moll, B. (2014). Knowledge growth and the allocation of time. *Journal of Political Economy*, 122(1), 1–51. 1

Morris, S. & Shin, H. S. (2002). Social value of public information. *American Economic Review*, 92(5), 1521–1534. 3

Nordhaus, W. D. (2007). A review of the stern review on the economics of climate change. *Journal of economic literature*, 45(3), 686–702. 6.1

Perla, J. & Tonetti, C. (2014). Equilibrium imitation and growth. *Journal of Political Economy*, 122(1), 52–76. 1

Soretz, S. (2007). Efficient dynamic pollution taxation in an uncertain environment. *Environmental and Resource Economics*, 36, 57–84. 1

Stockey, N. L. (1998). Are there limitations to growth? *International Economic Review*, 39(1), 1–31. 1

Traeger, C. P. (2015). Closed-form integrated assessment and uncertainty. *Working Paper*. 1, 6.1

Tsur, Y. & Zemel, A. (1998). Pollution control in an uncertain environment. *Journal of Economic Dynamics and Control*, 22, 967–975. 1

Woodford, M. & Walsh, C. E. (2005). *Interest and prices: Foundations of a theory of monetary policy*. Cambridge Univ Press. 12

Appendix

A Proof of lemma 1

Maximization of (8) with respect to I_{it} leads to the first-order condition

$$\frac{\partial \pi(x_{it}, I_{it}; x_t^\star)}{\partial I} + \delta_t \mathbb{E}\left[\left. \frac{\mathcal{V}(x_{it} + I_{it} + \tilde{\varepsilon}; x_{t+1}^\star)}{\partial x} \right| \omega_t, w_{it} \right] = 0 \tag{37}$$

while the envelop condition yields

$$\frac{\partial \mathcal{V}(x_{it}; x_t^\star)}{\partial x} = \frac{\partial \pi(x_{it}, I_{it}; x_t^\star)}{\partial x} + \delta_t \mathbb{E}\left[\left. \frac{\mathcal{V}(x_{it} + I_{it} + \tilde{\varepsilon}; x_{t+1}^\star)}{\partial x} \right| \omega_t, w_{it} \right] \tag{38}$$

implying

$$\frac{\partial \mathcal{V}(x_{it}; x_t^\star)}{\partial x} = \frac{\partial \pi(x_{it}, I_{it}; x_t^\star)}{\partial x} - \frac{\partial \pi(x_{it}, I_{it}; x_t^\star)}{\partial I} = x_t^\star - x_{it} + 1.$$

Plugging this expression in (37) evaluated in expectation for period $t+1$ yields

$$1 + r_t = \mathbb{E}\left[x_{t+1}^\star - (x_{it} + I_{it} + \tilde{\varepsilon}) + 1 | \omega_t, w_{it} \right] = \hat{x}_{it+1} - (x_{it} + I_{it}) + 1.$$

Reorganizing terms gives (9). Replacing in (3) and using (6), we obtain that firm i next period AGT index follows

$$x_{it+1} = x_{t+1}^\star + \tau \eta_t + (1-\tau)\nu_{it} - r_t + \varepsilon_{it+1}.$$

As $\int \nu_{it} di = \int \varepsilon_{it+1} di = 0$, we obtain

$$\mu_{t+1} = \int_i x_{it+1} di = x_{t+1}^\star + \tau \eta_t - r_t, \tag{39}$$

and thus

$$x_{it+1} = \mu_{t+1} + (1-\tau)\nu_{it} + \varepsilon_{it+1}.$$

As idiosyncratic investments depend on the firms' current technologies and on signals that are normally distributed, their next period technologies are also normally distributed around μ_{it+1} with variance

$$\mathbb{V}[x_{it+1}] = \sigma_\varepsilon^2 + (1-\tau)^2 \sigma_\nu^2.$$

Using (39) to substitutes for $x_{t+1}^\star$ in (5) yields

$$\mu_{t+1} - (\tau \eta_t - r_t) = \mu_t + \lambda(\mu_{t+1} - \mu_t) + z_t + \varepsilon_t^\star$$

which gives (10).

B Proof of lemma 2

Using (1), (11) and (12), we have

$$\tilde{y}_{t+1} = \tilde{c}_{t+1} + \rho e_{t+1} = gq_t + g_0 + \tilde{\kappa}_{t+1} - (\tilde{\mu}_{t+2} - \mu_{t+1}) + \rho(\theta e_t + \xi\mu_t - \varphi q_t + \hat{e}) \quad (40)$$

in which, using (16),

$$\tilde{\mu}_{t+2} = a_1\mu_{t+1} + a_2(\theta e_t + \xi\mu_t - \varphi q_t + \hat{e}) + a_3(gq_t + g_0 + \tilde{\kappa}_{t+1}) + a_4 + a_5(\tau\tilde{\eta}_{t+1} + \tilde{\varepsilon}^{\star}_{t+1}) + Z_{t+1} \quad (41)$$

where Z_{t+1} is a function of the z_{t+h}, $h = 2, 3...$ As the resulting expression of $\tilde{y}_{t+1}$ is a linear combinaison of iid random shocks normally distributed, it is also normally distributed with variance σ_y given by (20) which is independent of t.

The coefficients $a_{j=1,\dots,6}$ and Z_t in (16) are derived as follows. Using $y_t = q_t - \mu_{t+1} + \mu_t + \rho e_t$ yields

$$\mathbb{E}_t[\tilde{y}_{t+1}] - y_t = g_0 - (1 - g + \rho\varphi)q_t - \mathbb{E}_t[\tilde{\mu}_{t+2}] + 2\mu_{t+1} - (1 - \rho\xi)\mu_t - \rho[(1-\theta)e_t - \hat{e}]$$

which gives, using (41) and collecting terms,

$$\begin{aligned}\mathbb{E}_t[\tilde{y}_{t+1}] - y_t \;=\;& g_0(1 - a_3) - [1 - (1 - a_3)g + (\rho - a_2)\varphi]q_t - (a_4 + Z_{t+1}) + (2 - a_1)\mu_{t+1} \\ & -[1 - (\rho - a_2)\xi]\mu_t - [\rho(1-\theta) + a_2\theta]e_t + (\rho - a_2)\hat{e}.\end{aligned}$$

Replacing in (15) gives

$$\begin{aligned} r_t \;=\;& \psi - \gamma^2\sigma_y^2/2 + \gamma\{[\varphi a_2 - (1 - g + \varphi\rho) - a_3 g]q_t - (a_1 - 2)\mu_{t+1} - [1 + \xi(a_2 - \rho)]\mu_t\} \\ & +\gamma\{g_0(1 - a_3) - e_t[a_2\theta + \rho(1-\theta)] - \hat{e}(a_2 - \rho) - a_4 - Z_{t+1}\}. \end{aligned} \quad (42)$$

Equalizing with (10), which can be rewritten as

$$r_t = z_t + \tau\eta_t + \varepsilon^{\star}_t - (1 - \lambda)(\mu_{t+1} - \mu_t), \quad (43)$$

and collecting terms yields

$$\begin{aligned}\mu_{t+1} \;=\;& \frac{1 - \lambda + \gamma[1 + \xi(a_2 - \rho)]}{1 - \lambda + \gamma(2 - a_1)}\mu_t + \gamma\frac{[a_2\theta + \rho(1-\theta)]e_t}{1 - \lambda + \gamma(2 - a_1)} + \gamma\frac{[1 - g + \varphi(\rho - a_2) + a_3 g]q_t}{1 - \lambda + \gamma(2 - a_1)} \\ & -\frac{\psi + \gamma g_0(1 - a_3) - \gamma\hat{e}(a_2 - \rho) - \gamma a_4 - \gamma^2\sigma_y^2/2}{1 - \lambda + \gamma(2 - a_1)} + \frac{\tau\eta_t + \varepsilon^{\star}_t}{1 - \lambda + \gamma(2 - a_1)} \\ & +\frac{z_t + \gamma Z_{t+1}}{1 - \lambda + \gamma(2 - a_1)}.\end{aligned}$$

Identifying with (16) gives

$$\begin{aligned} a_1 &= \frac{1-\lambda+\gamma[1+\xi(a_2-\rho)]}{1-\lambda+\gamma(2-a_1)}, a_2 = \frac{\gamma\rho(1-\theta)}{1-\lambda+\gamma(2-a_1-\theta)}, \\ a_3 &= \gamma\frac{1-g+\varphi(\rho-a_2)}{1-\lambda+\gamma(2-a_1-g)}, \\ a_4 &= \frac{\gamma[\hat{e}(a_2-\rho)+a_4+\gamma\sigma_y^2/2]-\psi-\gamma g_0(1-a_3)}{1-\lambda+\gamma(2-a_1)}, \\ a_5 &= \frac{1}{1-\lambda+\gamma(2-a_1)} \end{aligned} \quad (44)$$

and

$$Z_t = a_5(z_t+\gamma Z_{t+1}) = a_5\sum_{i=0}^{+\infty}(a_5\gamma)^i z_{t+i}. \quad (45)$$

The first two equations of (44) form a system involving only coefficients a_1 and a_2 that can be solved separately from the others. Observe also that $a_2 = \rho$ and $a_1 = a_1^0 \equiv 1 + (1-\lambda)/\gamma$ is a degenerate solution of this system. More precisely, using $1-\lambda+\gamma(2-a_1) = \gamma(a_1^0 - a_1 + 1)$, we can (44) as

$$\begin{aligned} a_1 &= \frac{a_1^0+\xi(a_2-\rho)}{a_1^0-a_1+1}, a_2 = \frac{\rho(1-\theta)}{a_1^0-a_1+1-\theta} \\ a_3 &= \frac{1-g+\varphi(\rho-a_2)}{a_1^0-a_1+1-g}, a_5 = \frac{1}{\gamma(a_1^0-a_1+1)} \\ a_4 &= \frac{\hat{e}(a_2-\rho)-g_0(1-a_3)+\gamma\sigma_y^2/2-\psi/\gamma}{a_1^0-a_1}. \end{aligned} \quad (46)$$

and we get, assuming $a_2 = \rho$ and $a_1 = a_1^0$ is solution, that $a_3 = 1$, $a_5 = 1/\gamma$ but a_4 diverges unless $\psi = \gamma^2\sigma_y^2/2 = (\tau^2\sigma_\eta^2 + \sigma_\star^2)/2$ in which case a_4 is indefinite. Alternative solutions can be derived as follows. From the expression of a_2, we get

$$a_2 - \rho = \frac{-\rho(a_1^0-a_1)}{a_1^0-a_1+1-\theta},$$

which, plugged into the expression for a_1, gives

$$a_1(a_1^0-a_1+1) - a_1^0 = \frac{-\rho\xi(a_1^0-a_1)}{a_1^0-a_1+1-\theta},$$

that can also be expressed as $(a_1^0 - a_1)P(a_1 - 1) = 0$ where $P(x) \equiv x(a_1^0 - \theta - x) + \rho\xi$ is a second degree polynomial. Non-degenerate solutions must solve $P(a_1 - 1) = 0$. As $P(0) = P(a_1^0 - \theta) = \rho\xi$ and $P(x)$ is concave, a_1 is either lower than 1 ($a_1 - 1$ is equal to the negative solution of $P(x) = 0$) or greater than $a_1^0 - \theta + 1 = 2 - \theta + (1-\lambda)/\gamma$ ($a_1 - 1$ is then equal to the positive solution of $P(x) = 0$). As z_t impacts positively μ_{t+1} for all t, it comes from (45) that $a_5 > 0$ which implies $a_1 - 1 < 1 + (1-\lambda)/\gamma$ and thus rules out the positive root of $P(a_1 - 1) = 0$. Consequently, $a_1 - 1$ corresponds to the negative solution of $P(x) = 0$ which is given by

$$x = \frac{1}{2}\left(a_1^0 - \theta - \sqrt{(a_1^0 - \theta)^2 + 4\rho\xi}\right).$$

After substituting $1 + (1-\lambda)/\gamma$ for a_1^0, we get

$$a_1 = 1 - \frac{1}{2}\left(\sqrt{[1 - \theta + (1-\lambda)/\gamma]^2 + 4\rho\xi} - 1 + \theta - (1-\lambda)/\gamma\right)$$

which gives (17). The condition $a_1 > 0$ can be expressed as

$$[3 - \theta + (1-\lambda)/\gamma]^2 - [1 - \theta + (1-\lambda)/\gamma]^2 - 4\rho\xi > 0$$

which simplifies to $2 + (1-\lambda)/\gamma > \theta + \rho\xi$. Using $a_1^0 - \theta - a_1 + 1 = \rho\xi/(1-a_1)$ in (46) gives

$$a_2 = \rho(1-\theta)(1-a_1)/\xi.$$

C Proof of lemma 3

Using (42), we get

$$\tilde{r}_t - \mathbb{E}_{t-1}[\tilde{r}_t] = \gamma\{[\varphi a_2 - (1 - g + \varphi\rho) - g a_3 - (a_1 - 2)a_3]\tilde{\kappa}_t - (a_1 - 2)a_5(\tau\tilde{\eta}_t + \tilde{\varepsilon}_t^\star)\}.$$

Using the expressions of a_3 and a_5 in (44), it can be expressed as

$$\tilde{r}_t - \mathbb{E}_{t-1}[\tilde{r}_t] = [1 - (1-\lambda)a_5](\tau\tilde{\eta}_t + \tilde{\varepsilon}_t^\star) - (1-\lambda)a_3\tilde{\kappa}_t, \tag{47}$$

that can also be derived from (43), which becomes

$$\tilde{r}_t = z_t + \tau\tilde{\eta}_t + \tilde{\varepsilon}_t^\star - (1-\lambda)(\tilde{\mu}_{t+1} - \mu_t),$$

using (16). The two-period-ahead wealth index is deduced from

$$\tilde{y}_{t+1} = \tilde{c}_{t+1} + \rho\tilde{e}_{t+1} = \tilde{q}_{t+1} + \tilde{\mu}_{t+1} - \tilde{\mu}_{t+2} + \rho\tilde{e}_{t+1}$$

where

$$\tilde{\mu}_{t+2} = a_1\tilde{\mu}_{t+1} + a_2\tilde{e}_{t+1} + a_3\tilde{q}_{t+1} + a_4 + a_5(\tau\tilde{\eta}_{t+1} + \tilde{\varepsilon}^\star_{t+1}) + Z_{t+1}.$$

Replacing leads to

$$\tilde{y}_{t+1} = (1-a_3)\tilde{q}_{t+1} + (1-a_1)\tilde{\mu}_{t+1} + (\rho - a_2)\tilde{e}_{t+1} - a_4 - a_5(\tau\tilde{\eta}_{t+1} + \tilde{\varepsilon}^\star_{t+1}) - Z_{t+1}$$

which gives

$$\begin{aligned}
\tilde{y}_{t+1} - \mathbb{E}[\tilde{y}_{t+1}] &= (1-a_3)(g\tilde{\kappa}_t + \tilde{\kappa}_{t+1}) + (1-a_1)[a_3\tilde{\kappa}_t + a_5(\tau\tilde{\eta}_t + \tilde{\varepsilon}^\star_t)] \\
&\quad +(a_2-\rho)\varphi\tilde{\kappa}_t - a_5(\tau\tilde{\eta}_{t+1} + \tilde{\varepsilon}^\star_{t+1}) \\
&= (1-a_3)\tilde{\kappa}_{t+1} - a_5(\tau\tilde{\eta}_{t+1} + \tilde{\varepsilon}^\star_{t+1}) + [(1-a_3)g + (1-a_1)a_3 \\
&\quad +(a_2-\rho)\varphi]\tilde{\kappa}_t + a_5(1-a_1)(\tau\tilde{\eta}_t + \tilde{\varepsilon}^\star_t)
\end{aligned}$$

and

$$\begin{aligned}
\mathbb{V}[\tilde{y}_{t+1}] &= (1-a_3)^2\sigma^2_\kappa + a_5^2(\tau^2\sigma^2_\eta + \sigma^2_\star) + [(1-a_3)g + (1-a_1)a_3 + (a_2-\rho)\varphi]^2\sigma^2_\kappa \\
&\quad +(1-a_1)^2a_5^2(\tau^2\sigma^2_\eta + \sigma^2_\star) \\
&= \sigma^2_y + [(1-a_3)g + (1-a_1)a_3 + (a_2-\rho)\varphi]^2\sigma^2_\kappa + (1-a_1)^2a_5^2(\tau^2\sigma^2_\eta + \sigma^2_\star) \\
&\equiv \sigma^2_{y_{+1}}.
\end{aligned}$$

D Proof of Lemma 4

At stationary state (μ_S, e_S, q_S), (16) simplifies to

$$\mu_S = \frac{a_2e_S + a_3q_S + a_4 + Z_S}{1-a_1} \tag{48}$$

where

$$Z_S = z_Sa_5\sum_{i=0}^{+\infty}(\gamma a_5)^i = \frac{a_5z_S}{1-\gamma a_5},$$

$q_S = g_0/(1-g)$, and from (12), $e_S = |\xi\mu_S - \varphi q_S + \hat{e}]/(1-\theta)$.

Substituting in (48) and using $a_2 = (1-a_1)(1-\theta)/\xi$, the multiplicative term affecting μ_S cancels out, and (48) simplifies to

$$z_S = \frac{1-\gamma a_5}{a_5}\left(a_2\frac{\varphi q_S - \hat{e}}{1-\theta} - a_3 q_S - a_4\right) \tag{49}$$

whatever μ_S. Using (44) to obtain $a_2 = \gamma a_5 \rho(1-\theta)/(1-\gamma\theta a_5), a_3 = -\gamma a_5[\varphi a_2 - (1-g+\varphi\rho) - a_3 g]$ and

$$a_4 = \gamma a_5\{[\hat{e}(a_2-\rho) + a_4 + \gamma\sigma_y^2/2] - \psi/\gamma - g_0(1-a_3)\},$$

allows us to get

$$\begin{aligned}
z_S &= (1-\gamma a_5)\gamma\left\{\frac{\rho(\varphi q_S - \hat{e})}{1-\gamma\theta a_5} + [\varphi(a_2-\rho) - (1-g) - a_3 g]q_S - \left[\hat{e}(a_2-\rho) + a_4 + \gamma\frac{\sigma_y^2}{2}\right]\right.\\
&\quad \left. + \frac{\psi}{\gamma} + g_0(1-a_3)\right\}\\
&= (1-\gamma a_5)\gamma\left\{\frac{\rho(\varphi q_S - \hat{e})}{1-\gamma\theta a_5} + (a_2-\rho)(\varphi q_S - \hat{e}) - (1-g+a_3 g)q_S - \left(a_4 + \gamma\frac{\sigma_y^2}{2}\right)\right.\\
&\quad \left. + \frac{\psi}{\gamma} + (1-g)q_S(1-a_3)\right\}\\
&= (1-\gamma a_5)\gamma\left\{(\varphi q_S - \hat{e})\left(\frac{\rho}{1-\gamma\theta a_5} + a_2 - \rho\right) - a_3 q_S - a_4 - \gamma\frac{\sigma_y^2}{2} + \frac{\psi}{\gamma}\right\},
\end{aligned}$$

where, using (49)

$$a_3 q_S + a_4 = a_2\frac{\varphi q_S - \hat{e}}{1-\theta} - \frac{z_S a_5}{1-\gamma a_5}.$$

We thus have

$$\begin{aligned}
z_S &= (1-\gamma a_5)\gamma\left\{(\varphi q_S - \hat{e})\left(\frac{\gamma\theta a_5\rho}{1-\gamma\theta a_5} - \frac{\theta a_2}{1-\theta}\right) + z_S\frac{a_5}{1-\gamma a_5} - \gamma\sigma_y^2/2 + \psi/\gamma\right\}\\
&= (1-\gamma a_5)\gamma\left(z_S\frac{a_5}{1-\gamma a_5} - \gamma\sigma_y^2/2 + \psi/\gamma\right)
\end{aligned}$$

hence $z_S = \psi - \gamma^2\sigma_y^2/2$.

E Proof of Proposition 1

The first-order condition with respect to ι_t is given by

$$\mathbb{E}\left[\frac{\partial u_t}{\partial c}\right] = \beta\mathbb{E}\left[\frac{\partial \mathcal{W}_{t+1}}{\partial \mu}\right] \tag{50}$$

where u_t and $\mathcal{W}_t$ are abbreviated notations for $u(c_t, e_t)$ and $\mathcal{W}(\mu_t, e_t, q_t)$ respectively. The envelop theorem gives

$$\frac{\partial \mathcal{W}_t}{\partial \mu} = \mathbb{E}\left[\frac{\partial u_t}{\partial c}\right] + \xi\beta\mathbb{E}\left[\frac{\partial \mathcal{W}_{t+1}}{\partial e}\right] + \beta\mathbb{E}\left[\frac{\partial \mathcal{W}_{t+1}}{\partial \mu}\right] \tag{51}$$

and

$$\frac{\partial \mathcal{W}_t}{\partial e} = \mathbb{E}\left[\frac{\partial u_t}{\partial e}\right] + \theta\beta\mathbb{E}\left[\frac{\partial \mathcal{W}_{t+1}}{\partial e}\right]. \tag{52}$$

Using (50), (51) can be written as

$$\frac{\partial \mathcal{W}_t}{\partial \mu} = 2\mathbb{E}\left[\frac{\partial u_t}{\partial c}\right] + \xi\beta\mathbb{E}\left[\frac{\partial \mathcal{W}_{t+1}}{\partial e}\right] \tag{53}$$

which, evaluated one period ahead in expectation gives

$$\mathbb{E}\left[\frac{\partial \mathcal{W}_{t+1}}{\partial \mu}\right] = 2\mathbb{E}\left[\frac{\partial u_{t+1}}{\partial c}\right] + \xi\beta\mathbb{E}\left[\frac{\partial \mathcal{W}_{t+2}}{\partial e}\right] = \frac{1}{\beta}\mathbb{E}\left[\frac{\partial u_t}{\partial c}\right]$$

using (50), hence

$$\mathbb{E}\left[\frac{\partial \mathcal{W}_{t+2}}{\partial e}\right] = \frac{1}{\xi\beta^2}\mathbb{E}\left[\frac{\partial u_t}{\partial c}\right] - \frac{2}{\xi\beta}\mathbb{E}\left[\frac{\partial u_{t+1}}{\partial c}\right].$$

Plugging this expression in (52) evaluated one period ahead yields

$$\mathbb{E}\left[\frac{\partial \mathcal{W}_{t+1}}{\partial e}\right] = \mathbb{E}\left[\frac{\partial u_{t+1}}{\partial e}\right] + \frac{\theta}{\xi\beta}\mathbb{E}\left[\frac{\partial u_t}{\partial c}\right] - \frac{2\theta}{\xi}\mathbb{E}\left[\frac{\partial u_{t+1}}{\partial c}\right]$$

We can thus express (53) as

$$\frac{\partial \mathcal{W}_t}{\partial \mu} = (2+\theta)\mathbb{E}\left[\frac{\partial u_t}{\partial c}\right] + \xi\beta\mathbb{E}\left[\frac{\partial u_{t+1}}{\partial e}\right] - 2\theta\beta\mathbb{E}\left[\frac{\partial u_{t+1}}{\partial c}\right]$$

which, evaluated one period ahead yield gives,

$$\mathbb{E}\left[\frac{\partial \mathcal{W}_{t+1}}{\partial \mu}\right] = (2+\theta)\mathbb{E}\left[\frac{\partial u_{t+1}}{\partial c}\right] + \xi\beta\mathbb{E}\left[\frac{\partial u_{t+2}}{\partial e}\right] - 2\theta\beta\mathbb{E}\left[\frac{\partial u_{t+2}}{\partial c}\right] = \frac{1}{\beta}\mathbb{E}\left[\frac{\partial u_t}{\partial c}\right]$$

using (50). Reorganizing terms, we obtain

$$\mathbb{E}\left[\frac{\partial u_t}{\partial c}\right] = (2+\theta)\beta\mathbb{E}\left[\frac{\partial u_{t+1}}{\partial c}\right] - 2\theta\beta^2\mathbb{E}\left[\frac{\partial u_{t+2}}{\partial c}\right] + \beta^2\xi\mathbb{E}\left[\frac{\partial u_{t+2}}{\partial e}\right]. \quad (54)$$

The relationship between the expected marginal utility of consumption from one period to the next is derived from the consumer's program. However, because the interest rate is unknown, (14) becomes

$$\mathbb{E}\left[\frac{\partial u(\tilde{c}_t, e_t)}{\partial c}\right] = \beta\mathbb{E}\left[(1+\tilde{r}_t^\star)\frac{\partial u(\tilde{c}_{t+1}, \tilde{e}_{t+1})}{\partial c}\right]$$

and we get, using (24),

$$\mathbb{E}\left[\frac{\partial u_t}{\partial c}\right] = (1+r_t^e)\beta\mathbb{E}\left[\frac{\partial u_{t+1}}{\partial c}\right].$$

Substituting in (54) for initial dates t and $t+1$ yields

$$\frac{\beta^2}{\delta_t^e\delta_{t+1}^e}\mathbb{E}\left[\frac{\partial u_{t+2}}{\partial c}\right] = \frac{(2+\theta)\beta^2}{\delta_{t+1}^e}\mathbb{E}\left[\frac{\partial u_{t+2}}{\partial c}\right] - 2\theta\beta^2\mathbb{E}\left[\frac{\partial u_{t+2}}{\partial c}\right] + \beta^2\xi\mathbb{E}\left[\frac{\partial u_{t+2}}{\partial e}\right]$$

where $\delta_t^e \equiv (1+r_t^e)^{-1}$, which upon simplifying and rearranging terms yields (23).

F Proof of Proposition 2

The dynamic (25) can be solved as follows. Defining $v_t = (r_t^e + c)^{-1}$, or equivalently $r_t^e = 1/v_t - c$, (25) becomes

$$\begin{aligned}\frac{1}{v_{t+1}} &= c + \frac{1/v_t - c - \rho\xi - 1 + \theta}{1 + \theta - 1/v_t + c} \\ &= c + \frac{1 - v_t(c + \rho\xi + 1 - \theta)}{v_t(c + 1 + \theta) - 1} \\ &= \frac{v_t[c(c+1+\theta) - c - \rho\xi - (1-\theta)] - c + 1}{v_t(c+1+\theta) - 1}\end{aligned}$$

which gives

$$v_{t+1} = \frac{v_t(c+1+\theta)-1}{v_t[c(c+\theta)-\rho\xi-1+\theta]-c+1},$$

an equation that simplifies to

$$\begin{aligned} v_{t+1} &= \frac{c+1+\theta}{1-c}v_t - \frac{1}{1-c} \\ &\equiv kv_t + k_0, \end{aligned} \tag{55}$$

under the conditions $c \neq 1$ and

$$c(c+\theta) = \rho\xi + 1 - \theta. \tag{56}$$

Provided that $k \neq 1$, the solution of the recurrence equation (55) is given by

$$v_{t+1} = k^{t+1}v_0 + k_0(1-k^{t+1})/(1-k)$$

which converges to $v_\infty = k_0/(1-k)$ if $|k| < 1$, i.e. if $1 > |(c+1+\theta)/(1-c)| > 0$, albeit with oscillations along its path if $k < 0$. The corresponding solution of (25) would converge to

$$r^e_\infty = (1-k)/k_0 - c = c + \theta$$

which must be equal to the solution of (26), i.e. we must have

$$r^e_\sharp = c + \theta \tag{57}$$

where $r^e_\sharp$ is either equal to $(A+\theta)/2 > 0$ or $-(A-\theta)/2 < 0$. Using (57) to substitute $r^e_\sharp - \theta$ for c in (56) yields (26). (56) is thus satisfied if (57) is true. Using (57) and (55) yields

$$k = \frac{r^e_\sharp + 1}{1+\theta-r^e_\sharp}. \tag{58}$$

We have $k > 0$ if $1+\theta > r^e_\sharp > -1$ (as we cannot have $r^e_\sharp < -1$ and $1+\theta-r^e_\sharp < 0$), and $0 < k < 1$ if $r^e_\sharp + 1 < 1+\theta-r^e_\sharp$, hence if $r^e_\sharp < \theta/2$ which rules out the positive root of (26). The condition $r^e_\sharp > -1$ leads to $-(A-\theta)/2 > -1$, hence $A < 2+\theta$. Squaring both sides and reorganizing terms yields

$$4\rho\xi < (2+\theta)^2 - (2-\theta)^2 = 8\theta$$

hence $2\theta > \rho\xi$. We have $k < 0$ if $r^e_\sharp < -1$ or if $r^e_\sharp > 1+\theta$. The positive root of (26) is ruled out since if $r^e_\sharp > 1+\theta$, the conditions $-1 < k < 0$ would imply $r^e_\sharp + 1 < r^e_\sharp - (1+\theta)$, hence $1 < -(1+\theta)$, an impossibility. If $r^e_\sharp < -1$, which implies $2\theta < \rho\xi$, we have $-1 < k < 0$ if $1 - r^e_\sharp < 1 - r^e_\sharp + \theta$ which is always true since $\theta > 0$. Substituting $-(A-\theta)/2$ for $r^e_\sharp$ in (58) yields

$$k = \frac{r^e_\sharp + 1}{1+\theta - r^e_\sharp} = \frac{2-(A-\theta)}{2(1+\theta)-\theta+A} = \frac{2+\theta-A}{2+\theta+A}.$$

From (57), the condition $c \neq 1$ is equivalently stated as $r^e_\sharp - \theta = -(\theta + A)/2 \neq 1$ which is always the case since both θ and A are positive.

Using as initial value $r^e_{t_0} = 1/v_{t_0} - c$, the solution of (25) at period $t \geq t_0$ is given by

$$r^e_t = \frac{(1-k)(r^e_{t_0}+c)}{(1-k)k^{t-t_0} + k_0(1-k^{t-t_0})(r^e_{t_0}+c)} - c$$

where $(1-k)/k_0 = r^e_\sharp + c$, which gives

$$\begin{aligned} r^e_t &= \frac{(r^e_\sharp + c)(r^e_{t_0}+c)}{(r^e_\sharp + c)k^{t-t_0} + (1-k^{t-t_0})(r^e_{t_0}+c)} - c \\ &= r^e_\sharp + \frac{(r^e_\sharp + c)(r^e_{t_0} - r^e_\sharp)k^{t-t_0}}{r^e_\sharp + c + (1-k^{t-t_0})(r^e_{t_0} - r^e_\sharp)}. \end{aligned}$$

Using (57) and (27), we obtain $r^e_\sharp + c = 2r^e_\sharp - \theta = -A$. Substituting allows us to obtain (28) which converges to $r^e_\sharp$ when $t \to +\infty$ if $r^e_{t_0} - r^e_\sharp \neq A$. Moreover, in the case $k > 0$, we have $r^e_t > r^e_\sharp$ for all $t \geq t_0$ if $A > (1 - k^{t-t_0})(r^e_{t_0} - r^e_\sharp)$ for all $t \geq t_0$, hence $A > r^e_{t_0} - r^e_\sharp$.

The covariance term in (24) is derived using $u(y_t) = -e^{-\gamma y_t}$ and $\mathbb{E}[e^{-\gamma\tilde{y}}] = e^{-\gamma(\mathbb{E}[\tilde{y}] - \gamma\mathbb{V}[\tilde{y}]/2)}$ which gives

$$\frac{u'(\tilde{y}_{t+1})}{\mathbb{E}_{t-1}\left[u'(\tilde{y}_{t+1})\right]} = e^{-\gamma(\tilde{y}_{t+1} - \mathbb{E}_{t-1}[\tilde{y}_{t+1}] + \gamma\sigma^2_{y+1}/2)}.$$

Consequently,

$$\begin{aligned} \frac{Cov_{t-1}\left(\tilde{r}_t, u'(\tilde{y}_{t+1})\right)}{\mathbb{E}_{t-1}\left[u'(\tilde{y}_{t+1})\right]} &= \mathbb{E}\left[(\tilde{r}_t - \mathbb{E}_{t-1}[\tilde{r}_t])\left(\frac{u'(\tilde{y}_{t+1})}{\mathbb{E}_{t-1}\left[u'(\tilde{y}_{t+1})\right]} - 1\right)\right] \\ = \mathbb{E}\left[\{[1-(1-\lambda)a_5]a_5(\tau\tilde{\eta}_t + \tilde{\varepsilon}^\star_t) - (1-\lambda)a_3\tilde{\kappa}_t\}\left(e^{-\gamma(\tilde{y}_{t+1} - \mathbb{E}_{t-1}[\tilde{y}_{t+1}] + \gamma\sigma^2_{y+1}/2)} - 1\right)\right] \\ = e^{-\gamma^2\sigma^2_{y+1}/2}\{[1-(1-\lambda)a_5]\mathbb{E}\left[(\tau\tilde{\eta}_t + \tilde{\varepsilon}^\star_t)e^{-\gamma(\tilde{y}_{t+1} - \mathbb{E}[\tilde{y}_{t+1}])}\right] \\ - (1-\lambda)a_3\mathbb{E}\left[\tilde{\kappa}_t e^{-\gamma(\tilde{y}_{t+1} - \mathbb{E}_{t-1}[\tilde{y}_{t+1}])}\right]\} \end{aligned}$$

where the last term can be written as

$$\mathbb{E}\left[\tilde{\kappa}_t e^{-\gamma(\tilde{y}_{t+1}-\mathbb{E}_{t-1}[\tilde{y}_{t+1}])}\right] = \mathbb{E}[\tilde{\kappa}_t e^{-\gamma[(1-a_3)g+(1-a_1)a_3+(a_2-\rho)\varphi]\tilde{\kappa}_t}] \times\mathbb{E}[e^{-\gamma[(1-a_3)\tilde{\kappa}_{t+1}+(1-a_1)a_5(\tau\tilde{\eta}_t+\tilde{\varepsilon}^\star_t)-a_5(\tau\tilde{\eta}_{t+1}+\tilde{\varepsilon}^\star_{t+1})]}$$

from independence. Using $\mathbb{E}[e^{-\gamma\tilde{X}}] = e^{-\gamma(\mathbb{E}\tilde{X}-\gamma\sigma_X^2/2)}$ for a normal random variable $\tilde{X}$, it comes that the last term is equal to $e^{\gamma^2\{\sigma_y^2+(1-a_1)^2a_5^2(\tau^2\sigma_\eta^2+\sigma_\star^2)\}/2}$. Moreover, using

$$\mathbb{E}\left[\tilde{X}e^{-\gamma\tilde{X}}\right] = -\frac{d}{d\gamma}\mathbb{E}[e^{-\gamma\tilde{X}}] = -\frac{d}{d\gamma}e^{-\gamma(\mathbb{E}\tilde{X}-\gamma\sigma_X^2/2)} = (\mathbb{E}\tilde{X}-\gamma\sigma_X^2)e^{-\gamma(\mathbb{E}\tilde{X}-\gamma\sigma_X^2/2)},$$

we get

$$\mathbb{E}[\tilde{\kappa}_t e^{-\gamma[(1-a_3)g+(1-a_1)a_3]\tilde{\kappa}_t}] = -\gamma[(1-a_3)g+(1-a_1)a_3+(a_2-\rho)\varphi]^2\sigma_\kappa^2 e^{\gamma^2[(1-a_3)g+(1-a_1)a_3]^2\sigma_\kappa^2/2}$$

which gives

$$\mathbb{E}\left[\tilde{\kappa}_t e^{-\gamma(\tilde{y}_{t+1}-\mathbb{E}_{t-1}[\tilde{y}_{t+1}])}\right] = -\gamma[(1-a_3)g+(1-a_1)a_3+(a_2-\rho)\varphi]^2\sigma_\kappa^2 e^{\gamma^2\sigma_{y_{t+1}}^2/2}$$

Similarly, we have

$$\begin{aligned}\mathbb{E}\left[(\tau\tilde{\eta}_t+\tilde{\varepsilon}^\star_t)e^{-\gamma(\tilde{y}_{t+1}-\mathbb{E}\tilde{y}_{t+1})}\right] &= \mathbb{E}\left[(\tau\tilde{\eta}_t+\tilde{\varepsilon}^\star_t)e^{-\gamma a_5(1-a_1)(\tau\tilde{\eta}_t+\tilde{\varepsilon}^\star_t)}\right] \\ &\quad\times\mathbb{E}[e^{-\gamma\{(1-a_3)\tilde{\kappa}_{t+1}+[(1-a_3)g+(1-a_1)a_3]\tilde{\kappa}_t-a_5(\tau\tilde{\eta}_{t+1}+\tilde{\varepsilon}^\star_{t+1})\}}] \\ &= -\gamma(1-a_1)^2a_5^2(\tau^2\sigma_\eta^2+\sigma_\star^2)e^{\gamma^2\sigma_{y_{t+1}}^2/2}.\end{aligned}$$

Collecting terms, we get

$$\begin{aligned}\frac{cov\,(\tilde{r}_t, u'(\tilde{y}_{t+1}))}{\mathbb{E}_{t-1}\left[u'(\tilde{y}_{t+1})\right]} &= (1-\lambda)a_3\gamma[(1-a_3)g+(1-a_1)a_3+(a_2-\rho)\varphi]^2\sigma_\kappa^2 \\ &\quad -[1-(1-\lambda)a_5]\gamma(1-a_1)^2a_5^2(\tau^2\sigma_\eta^2+\sigma_\star^2) \\ &= \gamma[(1-\lambda)a_3(\sigma_{y+1}^2-\sigma_y^2)-(1-a_1)^2a_5^2(\tau^2\sigma_\eta^2+\sigma_\star^2)].\end{aligned}$$

which gives (30).

G Proof of Proposition 3

This result can be obtained recursively by observing that (28) can be rewritten in term of normalized gaps as $d_{t_0+h} = f^h(d_{t_0})$ with $f^0(x) = x$ and

$f^h(x) \equiv f \circ f^{h-1}(x)$ for all $h \geq 1$. Indeed, it is true for $h = 1$ since $d_{t_0+1} = f(d_{t_0})$ and supposing it is true for $t+h$, it is true for $t+h+1$: we have $f(d_{t+h}) = f(f^h(d_t)) = f^{h+1}(d_t) = d_{t+h+1}$. A direct and alternative proof is obtained using $f^h(x) = k^h x/[1-(1-k^h)x]$: we get

$$\begin{aligned} f(f^h(x)) &= \frac{kf^h(x)}{1-(1-k)f^h(x)} = \frac{k^{h+1}x}{1-(1-k^h)x-(1-k)k^h x} = \frac{k^{h+1}x}{1-(1-k^{h+1})x} \\ &= f^{h+1}(x). \end{aligned}$$

The condition $d_{t_0} < 1$ is derived from $A > r^e_{t_0} - r^e_\sharp$ in proposition 2.

H Proof of Proposition 4

Using (10) recursively, we get

$$\mathbb{E}_{t_0}[\tilde{\mu}_{t_0+h}] = \mu_{t_0} - \sum_{i=0}^{h-1} \frac{\mathbb{E}_{t_0}[\tilde{r}^\star_{t_0+i}]}{1-\lambda} = \mu_{t_0} - \frac{hr^\star_\sharp + A\sum_{i=0}^{h-1} f^i(d_{t_0})}{1-\lambda}.$$

and we have

$$\tilde{\mu}_{t_0+h} = \mathbb{E}_{t_0}[\tilde{\mu}_{t_0+h}] + \sum_{i=0}^{h-1} \frac{\tau\tilde{\eta}_{t_0+i} + \tilde{\varepsilon}^\star_{t_0+i} - A\tilde{v}_{t_0+i}}{1-\lambda}.$$

Since the disturbances $\tilde{v}_t$ are iid, we obtain using $\tilde{v}_t = (\tilde{r}_t - \mathbb{E}[\tilde{r}_t])/A$ and (47) that $\mathbb{V}_{t_0}[\tilde{\mu}_{t_0+h}] = h\mathbb{V}[\tilde{\mu}_{t+1}]$ where $\mathbb{V}[\tilde{\mu}_{t+1}] = a_3^2\sigma_\kappa^2 + a_5^2(\tau^2\sigma_\eta^2 + \sigma_\star^2)$.

By definition of T,

$$\mathbb{E}_{t_0}[\tilde{\mu}_{t_0+T}] - \mu_M = \mu_{t_0} - \mu_M - \frac{Tr^\star_\sharp + \sum_{i=0}^{T} f^i(d_{t_0})}{1-\lambda} \approx 0$$

and thus

$$-Tr^\star_\sharp - \sum_{i=0}^{T} f^i(d_{t_0}) \approx (1-\lambda)(\mu_M - \mu_{t_0})$$

where

$$\sum_{i=0}^{T} f^i(d_{t_0}) = d_{t_0}\sum_{i=0}^{T} \frac{k^i}{1-(1-k^i)d_{t_0}} < \frac{d_{t_0}}{1-d_{t_0}}\sum_{i=0}^{T} k^i = \frac{d_{t_0}}{1-d_{t_0}}\frac{1-k^{T+1}}{1-k} < \frac{d_{t_0}}{1-d_{t_0}}\frac{1}{1-k}$$

hence

$$T < \frac{1}{-r^\star_\sharp}\left\{(1-\lambda)(\mu_M - \mu_{t_0}) + \frac{d_{t_0}}{1-d_{t_0}}\frac{1}{1-k}\right\}.$$

The environmental forecast at horizon h is derived from (12) which can be rewritten

$$e_S - \mathbb{E}[\tilde{e}_{t+h}] = \theta(e_S - \mathbb{E}[\tilde{e}_{t+h-1}]) + \xi(\mu_S - \mathbb{E}[\tilde{\mu}_{t+h-1}]) - \varphi(q_S - \mathbb{E}[q_{t+h-1}])$$

where $1 \leq h < T$. Solving the recursion yields

$$e_S - \mathbb{E}[\tilde{e}_{t+h}] = \theta^h(e_S - e_t) + \xi\sum_{i=0}^{h-1}\theta^i(\mu_S - \mathbb{E}[\tilde{\mu}_{t+h-1-i}]) - \varphi\sum_{i=0}^{h-1}\theta^i(q_S - \mathbb{E}[\tilde{q}_{t+h-1-i}]). \tag{59}$$

Using

$$\mu_S - \mathbb{E}[\mu_{t+h-1-i}] = \mu_S - \mu_t + \frac{(h-1-i)r^\star_\sharp}{1-\lambda} + \frac{A}{1-\lambda}\sum_{j=0}^{h-2-i} d_{t+j}$$

we obtain that the expected value of the environmental gap at horizon $h \geq 1$ is given by:

$$\begin{aligned} e_S - \mathbb{E}[\tilde{e}_{t+h}] \quad = \quad & \theta^h(e_S - e_t) + \xi(\mu_S - \mu_t)\sum_{i=0}^{h-1}\theta^i + \frac{\xi r^\star_\sharp}{1-\lambda}\sum_{i=0}^{h-1}\theta^i(h-1-i) \\ & + \frac{A\xi}{1-\lambda}\sum_{i=0}^{h-1}\theta^i\sum_{j=0}^{h-2-i} d_{t+j} - \varphi(q_S - q_t)\sum_{i=0}^{h-1}\theta^i g^{h-1-i} \end{aligned}$$

where

$$\sum_{i=0}^{h-1}\theta^i(h-1-i) = (h-1)\sum_{i=0}^{h-1}\theta^i - \sum_{i=0}^{h-1}\theta^i i$$

with

$$\sum_{i=0}^{h-1}\theta^i i = \theta\frac{d}{d\theta}\left[\sum_{i=0}^{h-1}\theta^i\right] = \theta\frac{d}{d\theta}\left[\frac{1-\theta^h}{1-\theta}\right] = \frac{\theta(1-\theta^h) - h\theta^h(1-\theta)}{(1-\theta)^2}$$

We thus have

$$\sum_{i=0}^{h-1}\theta^i(h-1-i) = \frac{(h-1)(1-\theta^h)(1-\theta)-\theta(1-\theta^h)+h\theta^h(1-\theta)}{(1-\theta)^2}$$
$$= \frac{(h-1)(1-\theta)-\theta(1-\theta^h)+\theta^h(1-\theta)}{(1-\theta)^2}$$
$$= \frac{(h-1)(1-\theta)-\theta+\theta^h}{(1-\theta)^2} = \frac{h(1-\theta)-1+\theta^h}{(1-\theta)^2}, \quad (60)$$

and thus

$$e_S - \mathbb{E}[\tilde{e}_{t+h}] = \theta^h(e_S-e_t)+\xi\,(\mu_S-\mu_t)\,\frac{1-\theta^h}{1-\theta} - \frac{\xi r^\star_\sharp}{1-\lambda}\frac{h(1-\theta)-1+\theta^h}{(1-\theta)^2}$$
$$-\frac{A\xi}{1-\lambda}\sum_{i=1}^{h-1}\theta^i\sum_{j=1}^{h-1-i}d_{t+j}-\varphi(q_S-q_t)\frac{g^h-\theta^h}{g-\theta}.$$

From (59) we also get

$$\tilde{e}_{t+h} = \mathbb{E}[\tilde{e}_{t+h}] + \xi\sum_{i=0}^{h-1}\theta^i\sum_{j=0}^{h-2-i}\left(\frac{\tau\tilde{\eta}_{t+j}+\tilde{\varepsilon}^\star_{t+j}-A\tilde{v}_{t+j}}{1-\lambda}\right) - \varphi\sum_{i=0}^{h-1}\theta^i\tilde{\kappa}_{t+h-1-i}$$

which gives, by definition of $\tilde{\nu}_t$

$$\tilde{e}_{t+h} = \mathbb{E}[\tilde{e}_{t+h}] + \xi\sum_{i=0}^{h-1}\theta^i\sum_{j=0}^{h-2-i}\left(a_5(\tau\tilde{\eta}_{t+j}+\tilde{\varepsilon}^\star_{t+j})+a_3\tilde{\kappa}_{t+j}\right) - \varphi\sum_{i=0}^{h-1}\theta^i\tilde{\kappa}_{t+h-1-i}$$
$$= \mathbb{E}[\tilde{e}_{t+h}] + a_5\xi\sum_{i=0}^{h-1}(\tau\tilde{\eta}_{t+h-1-i}+\tilde{\varepsilon}^\star_{t+h-1-i})\sum_{j=0}^{i-1}\theta^j + \sum_{i=0}^{h-1}\left(-\varphi\theta^i + a_3\xi\sum_{j=0}^{i-1}\theta^j\right)\kappa_{t+h-1-i}$$

which follows a normal distribution with variance:

$$\mathbb{V}[\tilde{e}_{t+h}] = a_5^2\xi^2(\tau^2\sigma_\eta^2+\sigma_\varepsilon^2)\sum_{i=0}^{h-1}\left(\frac{1-\theta^i}{1-\theta}\right)^2 + \sigma_\kappa^2\sum_{i=0}^{h-1}\left(-\varphi\theta^i + a_3\xi\frac{1-\theta^i}{1-\theta}\right)^2$$

The policy schedule can be derived from

$$z_{t_0+h} = \mathbb{E}[\tilde{r}_{t_0+h}] - r^\star_\sharp - Ad_{t_0+h}$$

where, for all $t \geq t_0$, we have from (15)

$$\mathbb{E}[\tilde{r}_t] = \psi + \gamma(\mathbb{E}[\tilde{y}_{t+1}] - \mathbb{E}[\tilde{y}_t]) - \gamma^2\sigma_{y+1}^2.$$

Using $\mathbb{E}[\tilde{y}_t] = \mathbb{E}[\tilde{c}_t] + \rho e_t$ and from (1) and (12) which gives

$$\begin{aligned}
\mathbb{E}[\tilde{r}_t] &= \psi - \gamma^2\sigma_{y+1}^2 + \gamma(\mathbb{E}[q_{t+1} - \mu_{t+2} + \mu_{t+1} + \rho e_{t+1}] - \mathbb{E}[q_t - \mu_{t+1} + \mu_t + \rho e_t]) \\
&= \psi - \gamma^2\sigma_{y+1}^2 + \gamma g(gq_{t-1} + g_0) + \gamma g_0 \\
&+ \gamma(2\mathbb{E}[\mu_{t+1}] + \rho\mathbb{E}[e_{t+1}] - \mathbb{E}[\mu_{t+2}] - gq_{t-1} - g_0 - \mu_t - \rho e_t) \\
&= \psi - \gamma^2\sigma_{y+1}^2 + \gamma g(gq_{t-1} + g_0) + \gamma g_0 + \gamma\left(\frac{\mathbb{E}_{t+1}[r_{t+1}^\star]}{1-\lambda} - \frac{\mathbb{E}_t[r_t^\star]}{1-\lambda}\right) \\
&+ \gamma\rho[\theta e_t - \varphi(gq_{t-1} + g_0) + \xi\mu_t + \hat{e} - gq_{t-1} - g_0 - \rho e_t] \\
&= \psi - \gamma^2\sigma_{y+1}^2 + \gamma\left(\frac{\mathbb{E}_{t+1}[r_{t+1}^\star]}{1-\lambda} - \frac{\mathbb{E}_t[r_t^\star]}{1-\lambda}\right) + \gamma(\rho(\theta-1)e_t \\
&+ \xi\rho\mu_t + (g - \varphi\rho - 1)gq_{t-1} + (g - \rho\varphi)g_0 + \rho\hat{e})
\end{aligned}$$

we get (35) after collecting terms.

I Approximation of $f(d_t + v_t)$

We have

$$\begin{aligned}
f(d_t) + f(v_t) &= \frac{kd_t}{1-(1-k)d_t} + \frac{kv_t}{1-(1-k)v_t} = \frac{k(d_t+v_t) - 2k(1-k)d_tv_t}{1-(1-k)(d_t+v_t)+(1-k)^2d_tv_t} \\
&= \frac{[1-(1-k)(d_t+v_t)]f(d_t+v_t) - 2k(1-k)d_tv_t}{1-(1-k)(d_t+v_t)+(1-k)^2d_tv_t} \\
&= f(d_t+v_t) - \frac{f(d_t+v_t)(1-k)^2d_tv_t + 2k(1-k)d_tv_t}{1-(1-k)(d_t+v_t)+(1-k)^2d_tv_t} \\
&= f(d_t+v_t) - \frac{(1-k)d_tv_t[f(d_t+v_t)(1-k)+2k]}{1-(1-k)(d_t+v_t)+(1-k)^2d_tv_t}
\end{aligned}$$

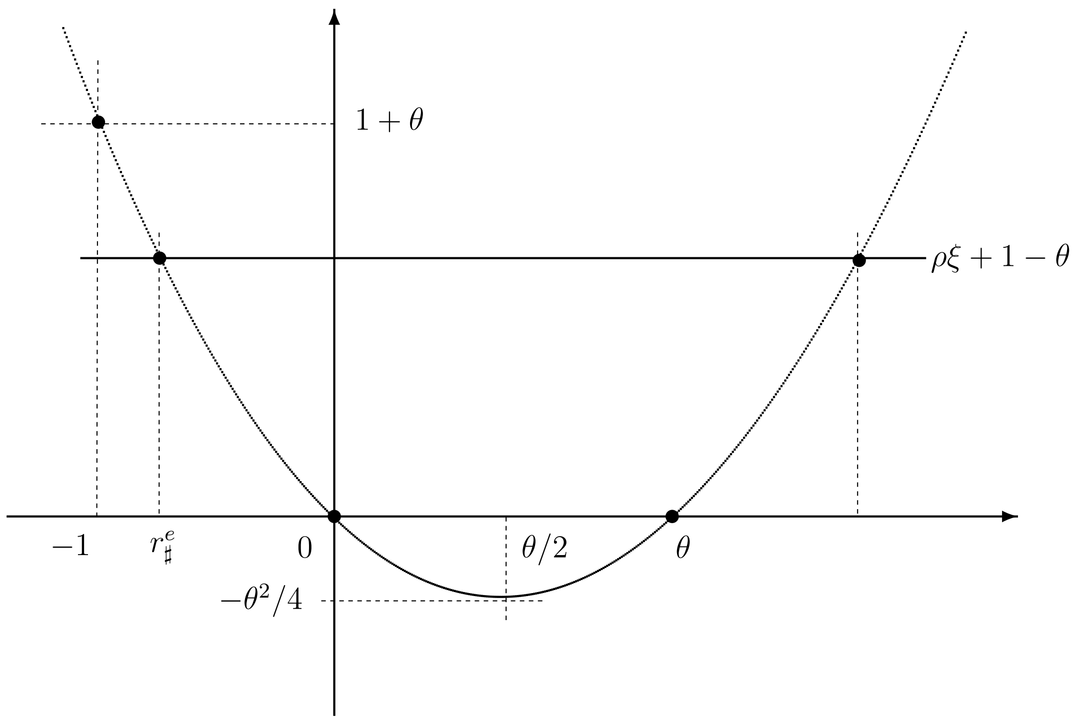


Figure 1: Long run optimal interest rate.

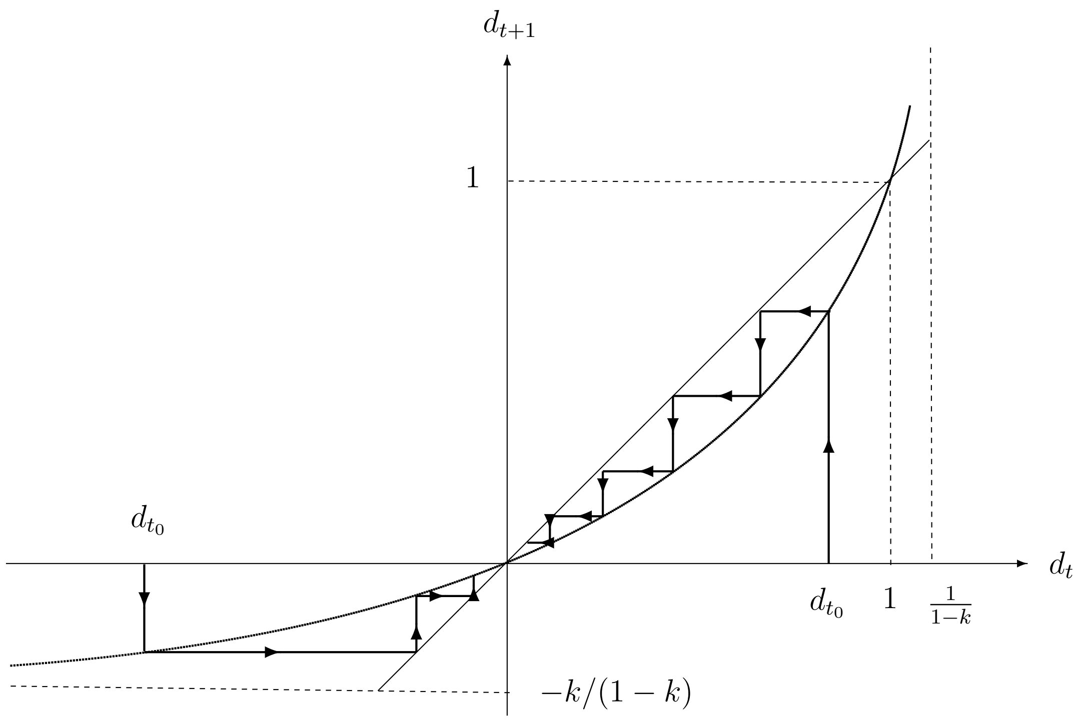


Figure 2: Convergence of the normalized gap.

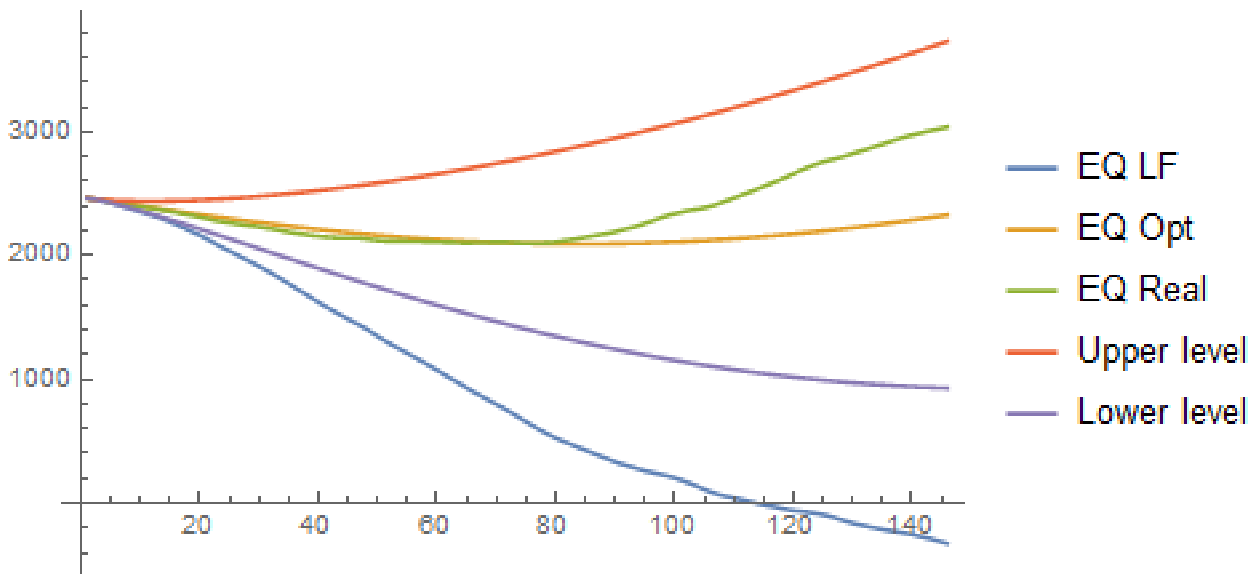


Figure 3: Environmental quality dynamics

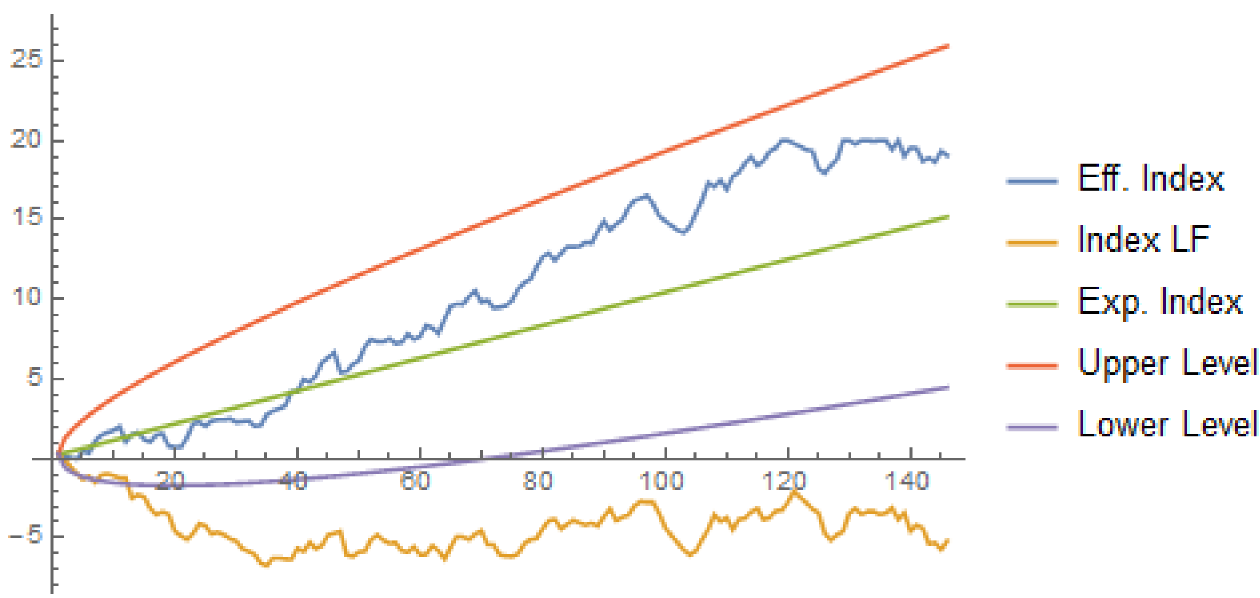


Figure 4: AGT Index Dynamics

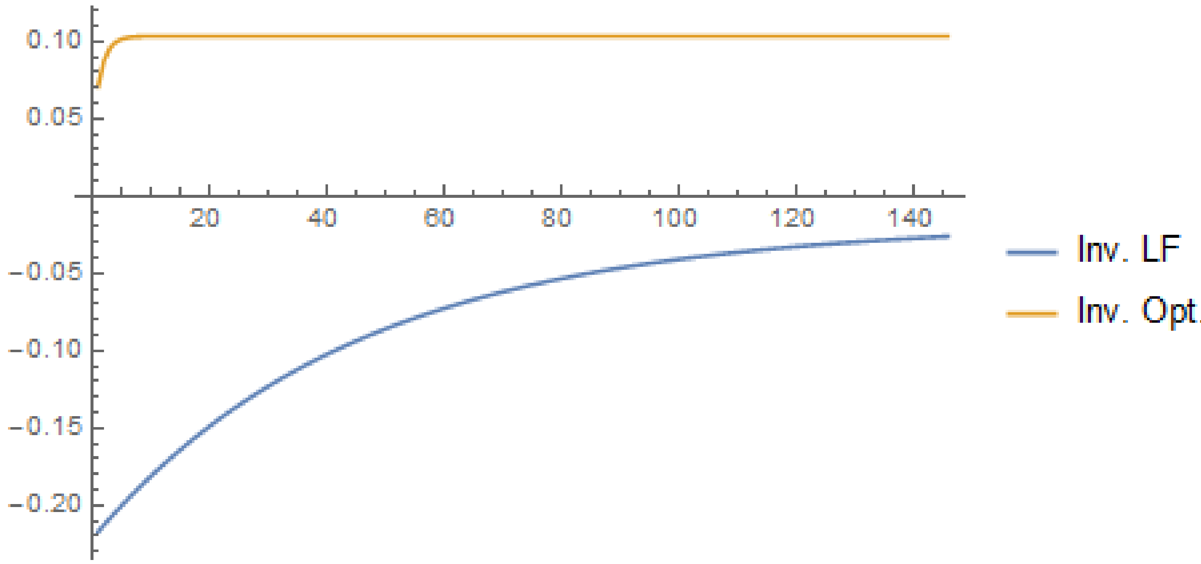


Figure 5: Investment level

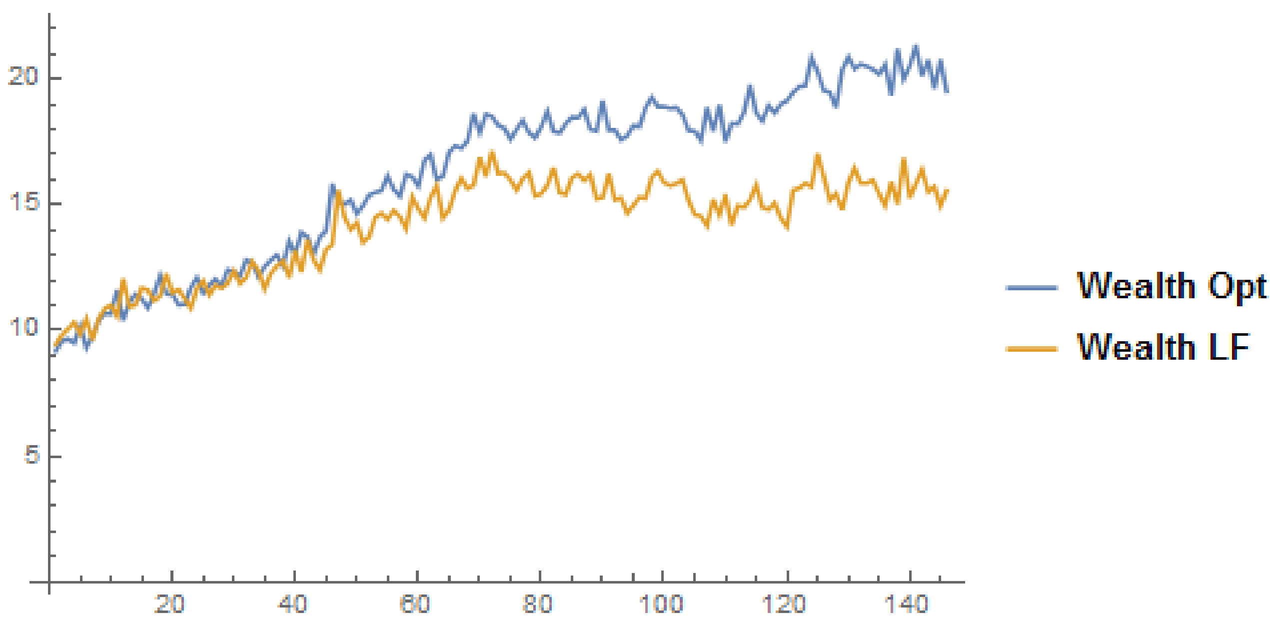


Figure 6: Welfare

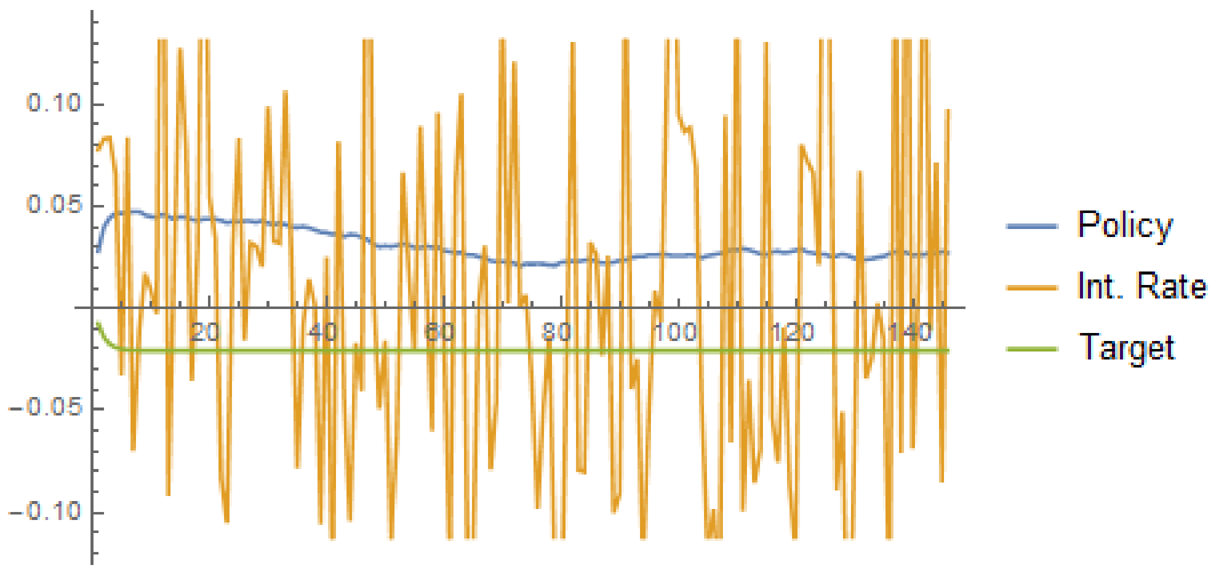
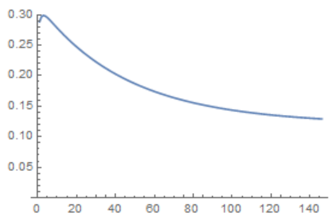
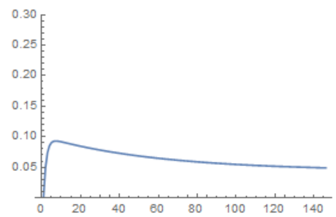


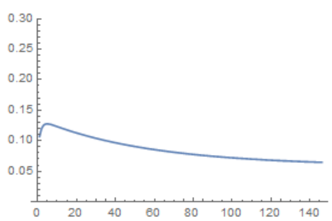
Figure 7: Interest rate



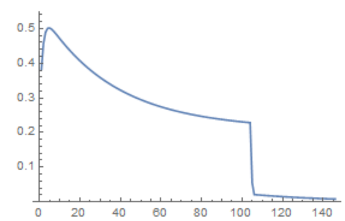
(a) Baseline ($\lambda = 0.8$)



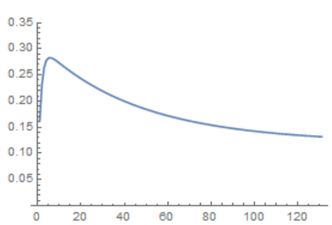
(b) $\lambda = 0.3$



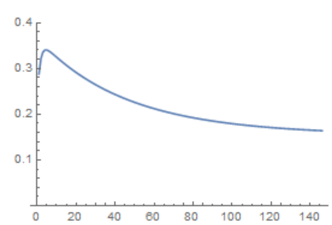
(c) $\lambda = 0.5$



(d) $\lambda = 0.9$



(e) $t_0 = 20$



(f) $\sigma_\kappa = \sigma_\eta = \sigma_\varepsilon = \sigma_\nu = 0.05$

Figure 8: Consumption differential between the regulated economy and laissez-faire