Cost Allocation in Natural Gas Distribution Networks

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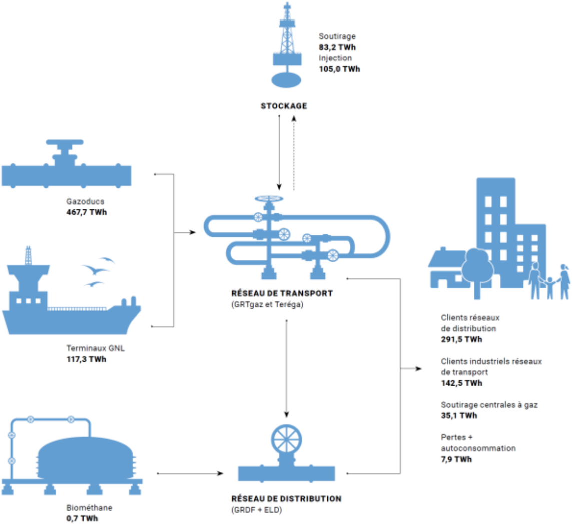
Gas distribution in France



Soutirage
83,2 TWh
Injection
105,0 TWh

STOCKAGE

Gazoducs
467,7 TWh

Terminaux GNL
117,3 TWh

Biométhane
0,7 TWh

RÉSEAU DE TRANSPORT
(GRTgaz et Teréga)

RÉSEAU DE DISTRIBUTION
(GRDF + ELD)

Clients réseaux
de distribution
291,5 TWh

Clients industriels réseaux
de transport
142,5 TWh

Soutirage centrales à gaz
35,1 TWh

Pertes +
autoconsommation
7,9 TWh

Gas distribution in France

In order to carry out its activity, a (gas distribution) network operator is faced with various **operation costs**:

- some costs are related to the extension of the network;
- others are related to network security;
- others are related to the maintenance of the network;
- etc.

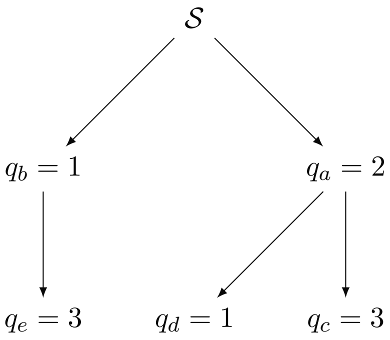
We want to evaluate the impact of consumer demands on operation costs.
How can these operation costs be allocated to consumers ?

Overview

1. Notations and definitions;
2. Optimistic design of a network;
3. The total cost of a network;
4. Normative approach to cost allocation rules;
5. Algorithmic approach to cost allocation rules;
6. Additional content.

The Model

The model



◇ $N = \{a, b, \ldots, n\}$ finite set of **consumers**.

◇ Consumers are connected to a source via **pipelines**, forming a **tree network** P.

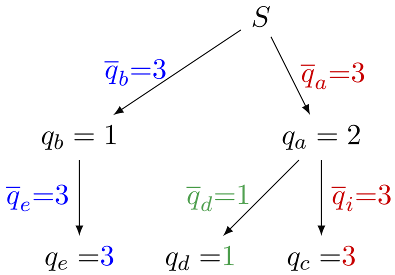
◇ Each $i \in N$ has an **effective demand** $q_i \in \mathbb{N}$, $q_i \leq K$.

¤ All effective demands are compiled in $q = (q_a, \ldots, q_n)$.
¤ The integer K serves as an upper bound for demands.

The model

◇ **Network design**: be able to satisfy any effective demand.

i.e. Each pipeline $i \in N$ meets its **effective capacity**— it can handle its highest downstream effective demand $\overline{q}_i$.



⌑ There exist alternatives to this design (not covered here).

The model

◇ A **Cost function** measures the cost of any pipeline of any capacity

$$C : N \times \{0, \ldots, K\} \to \mathbb{R}_+,$$

e.g. The cost of pipeline i sized at capacity j is $C(i, j) \in \mathbb{R}_+$.

C	a	b	c	d	e
1	5	2	7	4	5
2	10	8	13	9	11
3	15	12	16	13	15

⌑ $C(i, 0) = 0$ and $C(i, j) \leq C(i, j + 1)$.

The model

◇ **Incremental costs** are defined as

$$\forall i \in N, \forall j \leq K, \quad A_{ij}^{C} = C(i,j) - C(i,j-1).$$

C	a	b	c	d	e
1	5	2	7	4	5
2	10	8	13	9	11
3	15	12	16	13	15

$$\begin{aligned} A_{a3}^{C} &= C(a,3) - C(a,2) \\ &= 15 - 10 \\ &= 5. \end{aligned}$$

⌑ A_{ij}^{C} represents the (additional) operation costs induced by upgrading pipeline i from capacity $j-1$ to j.

The model

◇ The cost function and the **Matrix of incremental costs** are equivalent objects.

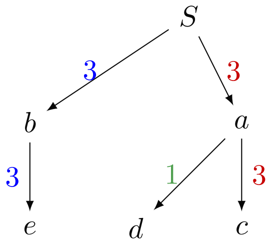
$$\forall i \in N, \forall j \leq K, \quad A^C_{ij} = C(i,j) - C(i,j-1).$$

C	a	b	c	d	e
1	5	2	7	4	5
2	10	8	13	9	11
3	15	12	16	13	15

A^C	a	b	c	d	e
1	5	2	7	4	5
2	5	6	6	5	5
3	5	4	3	4	5

The model

◇ The **total cost** of operating the network is computed as the sum of the costs of all pipelines, where each pipeline meets its effective capacity.



C	a	b	c	d	e
1	5	2	7	4	5
2	10	8	13	9	10
3	15	12	16	13	15

Total cost $= 62$.

◇ **Gas distribution (cost allocation) problem**: How to divide this total cost among consumers?

Cost Allocation Rules

Cost Allocation Rules

◇ A gas distribution problem is denoted by (q, A^C).

◇ To properly define rules, endow each consumer $i \in N$ with the discrete set of **demand units** $\{1, \ldots, q_i\}$.

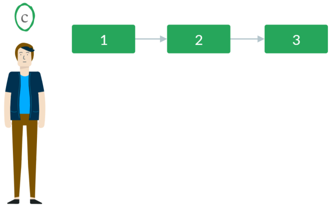


Figure: Demand units of consumer c

⌑ The class of all gas distribution problems is denoted by GDP.

Cost Allocation Rules

◇ A (cost allocation) rule is a map

$$\begin{aligned} f : GDP &\to \mathbb{R}_+^{|N|\times K} \\ (q, A^C) &\mapsto \begin{pmatrix} f_{a1} & \cdots & f_{n1} \\ \vdots & \ldots & \vdots \\ f_{aK} & \cdots & f_{nK} \end{pmatrix} \end{aligned}$$

⌑ Each coordinate $f_{ij}(q, A^C) \in \mathbb{R}_+$ captures the incremental allocation assigned to consumer i for an increase in demand from $j-1$ to j.

⌑ $f_{ij} = 0$ for each $j > q_i$.

Cost Allocation Rules

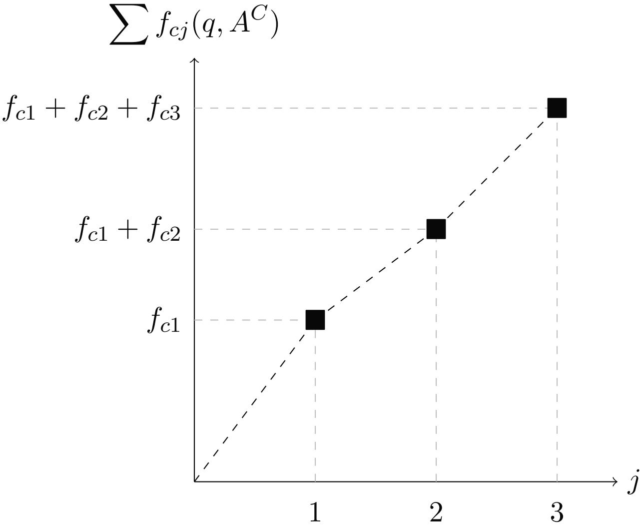
Recall that $q_a = 2$, $q_b = 1$, $q_c = 3$, $q_d = 1$ and $q_e = 3$.

$$\begin{pmatrix} f_{a1} & f_{b1} & f_{c1} & f_{d1} & f_{e1} \\ f_{a2} & 0 & f_{c2} & 0 & f_{e2} \\ 0 & 0 & f_{c3} & 0 & f_{e3} \end{pmatrix}$$

- The total amount charged to a consumer $i \in N$ is given by

$$F_i(q, A^C) = \sum_{j \leq q_i} f_{ij}(q, A^C).$$

Cost Allocation Rules



Normative approach based on principles.

Normative approach

◇ A **rule** satisfies the **Budget balanced principle** and the **Independence to higher demands principle**:

(i) **Budget balanced principle**: a rule recovers the total cost of operating the network.

(ii) **Independence to higher demands principle**: the amount allocated to a demand unit of a consumer is independent from any other greater demand unit.

Normative approach

◇ I propose three cost allocation rules:

- the Connection rule,
- the Uniform rule;
- and the Mixed rules.

◇ Each rule is in line with the **Budget balanced principle** and the **Independence to higher demands principle** (by definition).

◇ We introduce **two other principles** to highlight the differences between these three rules.

Normative approach

(iii) **Connection principle**: a consumer should only be charged for the costs associated with the specific pipelines that connect him to the source.

(iv) **Uniformity principle**: two consumers with the same demands should be charged the same amount regardless of their geographical location.

⌑ Clearly, the two principles are incompatible.

Normative approach

(i) **Budget balanced principle**

(ii) **Independence to higher demands principle**

(iii) **Connection principle**

(iv) Uniformity principle

$\Longrightarrow$ The **Connection rule**

Normative approach

(i) **Budget balanced principle**

(ii) **Independence to higher demands principle**

(iii) Connection principle

(iv) **Uniformity principle**

$\Longrightarrow$ The **Uniform rule**

Normative approach

(i) **Budget balanced principle**

(ii) **Independence to higher demands principle**

(iii) **Connection principle**

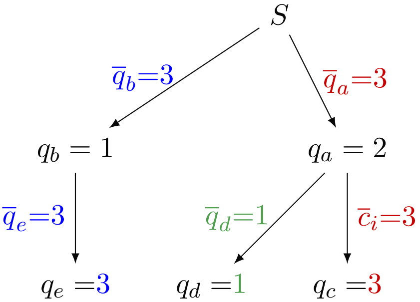
(iv) **Uniformity principle**

$\Longrightarrow$ The **Mixed rules**

Computing the rules

Computing the rules

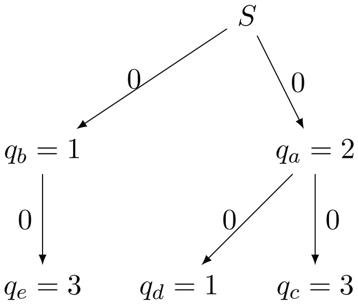
- ◇ **Network design**: be able to satisfy any effective demand.



- ◇ Let us build this network **step by step** to understand how the rules work.

Computing the rules

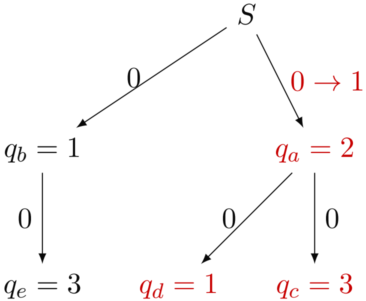
- **Step 0**: no network.



- No costs, which implies $\begin{array}{c} \\ 1 \\ 2 \\ 3 \end{array} \begin{array}{c} \begin{array}{ccccc} a & b & c & d & e \end{array} \\ \begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix} \end{array}$

Computing the rules

◇ **Step 1**: Upgrade a pipeline (let us choose a) capacity by one unit.



◇ This generates the incremental cost $\mathbf{A}^{\mathbf{C}}_{\mathbf{a1}}$.

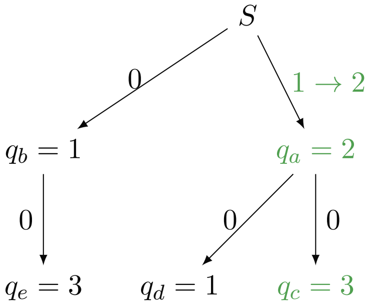
Computing the rules

Connection rule: $\mathbf{A^C_{a1}} \rightsquigarrow \begin{array}{c} \\ 1 \\ 2 \\ 3 \end{array} \begin{array}{c} \begin{array}{ccccc} a & b & c & d & e \end{array} \\ \begin{pmatrix} \mathbf{A^C_{a1}}/3 & 0 & \mathbf{A^C_{a1}}/3 & \mathbf{A^C_{a1}}/3 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix} \end{array}$

Uniform rule: $\mathbf{A^C_{a1}} \rightsquigarrow \begin{array}{c} \begin{array}{ccccc} a & b & c & d & e \end{array} \\ \begin{pmatrix} \mathbf{A^C_{a1}}/5 & \mathbf{A^C_{a1}}/5 & \mathbf{A^C_{a1}}/5 & \mathbf{A^C_{a1}}/5 & \mathbf{A^C_{a1}}/5 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix} \end{array}$

Computing the rules

◇ **Step 2**: Upgrade the same pipeline's capacity by one additional unit.



◇ This generates the incremental cost $\mathbf{A^{C}_{a2}}$.

Computing the rules

Connection rule: $\mathbf{A^{C}_{a2}} \curvearrowright$

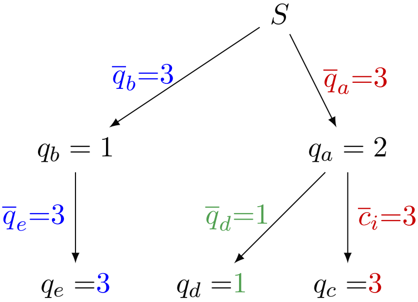
$$\begin{array}{c} \\ 1 \\ 2 \\ 3 \end{array} \begin{array}{c} \begin{array}{ccccc} a & b & c & d & e \end{array} \\ \begin{pmatrix} \mathbf{A^{C}_{a1}}/3 & 0 & \mathbf{A^{C}_{a1}}/3 & \mathbf{A^{C}_{a1}}/3 & 0 \\ \mathbf{A^{C}_{a2}}/2 & 0 & \mathbf{A^{C}_{a2}}/2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix} \end{array}$$

Uniform rule: $\mathbf{A^{C}_{a2}} \curvearrowright$

$$\begin{array}{c} \begin{array}{ccccc} a & b & c & d & e \end{array} \\ \begin{pmatrix} \mathbf{A^{C}_{a1}}/5 & \mathbf{A^{C}_{a1}}/5 & \mathbf{A^{C}_{a1}}/5 & \mathbf{A^{C}_{a1}}/5 & \mathbf{A^{C}_{a1}}/5 \\ \mathbf{A^{C}_{a2}}/3 & 0 & \mathbf{A^{C}_{a2}}/3 & 0 & \mathbf{A^{C}_{a2}}/3 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix} \end{array}$$

Computing the rules

◇ Continue until you recover the network as it is supposed to be designed.



⌑ Both the Connection rule and the Uniform rule can be computed in polynomial time.

Computing the rules

◇ The two rules lead to two different allocations.

$$\text{Connection rule} \rightarrow \quad \begin{pmatrix} Cr_{a1} & Cr_{b1} & Cr_{c1} & Cr_{d1} & Cr_{e1} \\ Cr_{a2} & 0 & Cr_{c2} & 0 & Cr_{e2} \\ 0 & 0 & Cr_{c3} & 0 & Cr_{e3} \end{pmatrix}$$

$$\text{Uniform rule} \rightarrow \quad \begin{pmatrix} Ur_{a1} & Ur_{b1} & Ur_{c1} & Ur_{d1} & Ur_{e1} \\ Ur_{a2} & 0 & Ur_{c2} & 0 & Ur_{e2} \\ 0 & 0 & Ur_{c3} & 0 & Ur_{e3} \end{pmatrix}$$

⌑ They reflect the connection principle and the uniformity principle, respectively.

Computing the rules

◇ For instance, in (q, A^C) where

A^C	a	b	c	d	e
1	5	2	7	4	5
2	5	6	6	5	5
3	5	4	3	4	5

$$\text{Connection rule} \rightarrow \begin{array}{c} \begin{array}{ccccc} a & b & c & d & e \end{array} \\ \begin{pmatrix} 1.7 & 1 & 8.7 & 5.7 & 6 \\ 2.5 & 0 & 8.5 & 0 & 11 \\ 0 & 0 & 8 & 0 & 9 \end{pmatrix} \end{array}$$

$$\text{Uniform rule} \rightarrow \begin{array}{c} \begin{array}{ccccc} a & b & c & d & e \end{array} \\ \begin{pmatrix} 4.6 & 4.6 & 4.6 & 4.6 & 4.6 \\ 7.3 & 0 & 7.3 & 0 & 7.3 \\ 0 & 0 & 8.5 & 0 & 8.5 \end{pmatrix} \end{array}$$

Computing the rules

◇ A Mixed rule is defined according to a (trade-off) system $(\alpha_1, \alpha_2, \ldots, \alpha_K)$, $\alpha_j \in [0,1]$ for each $j \in K$.

$$\begin{array}{l} \alpha^1 \times \\ \alpha^2 \times \\ \alpha^3 \times \end{array} \begin{pmatrix} Cr_{a1} & Cr_{b1} & Cr_{c1} & Cr_{d1} & Cr_{e1} \\ Cr_{a2} & 0 & Cr_{c2} & 0 & Cr_{e2} \\ 0 & 0 & Cr_{c3} & 0 & Cr_{e3} \end{pmatrix}$$

$$+ \begin{array}{l} (1-\alpha^1)\times \\ (1-\alpha^2)\times \\ (1-\alpha^3)\times \end{array} \begin{pmatrix} Ur_{a1} & Ur_{b1} & Ur_{c1} & Ur_{d1} & Ur_{e1} \\ Ur_{a2} & 0 & Ur_{c2} & 0 & Ur_{e2} \\ 0 & 0 & Ur_{c3} & 0 & Ur_{e3} \end{pmatrix}$$

$$= \begin{pmatrix} Mr_{a1} & Mr_{b1} & Mr_{c1} & Mr_{d1} & Mr_{e1} \\ Mr_{a2} & 0 & Mr_{c2} & 0 & Mr_{e2} \\ 0 & 0 & Mr_{c3} & 0 & Mr_{e3} \end{pmatrix}$$

⌑ Observe that $\alpha_j \neq \alpha_{j'}$, $j \neq j'$, is possible.

Computing the rules

Pick $\alpha = (1, 0.8, 0.5)$

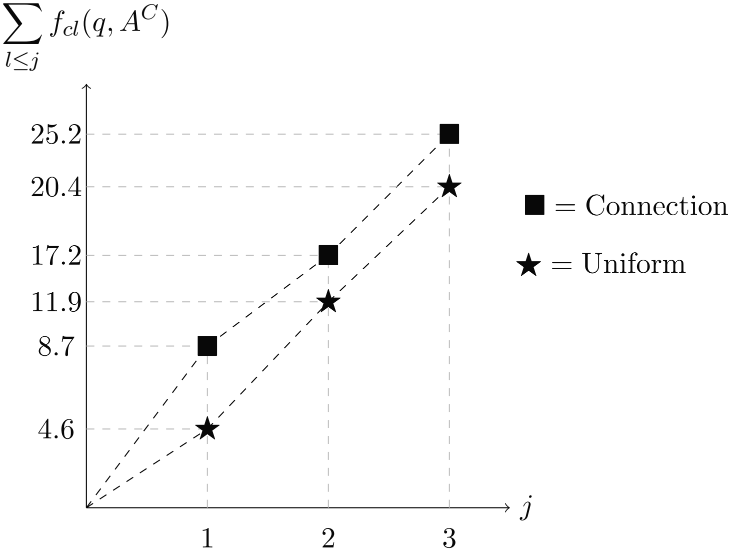
$$\text{Connection rule} \rightarrow \begin{array}{c} \\ 1\times \\ 0.8\times \\ 0.5\times \end{array} \begin{array}{c} \begin{array}{ccccc} a & b & c & d & e \end{array} \\ \begin{pmatrix} 1.7 & 1 & 8.7 & 5.7 & 6 \\ 2.5 & 0 & 8.5 & 0 & 11 \\ 0 & 0 & 8 & 0 & 9 \end{pmatrix} \end{array}$$

$$\text{Uniform rule} \rightarrow \begin{array}{c} \\ 0\times \\ 0.2\times \\ 0.5\times \end{array} \begin{array}{c} \begin{array}{ccccc} a & b & c & d & e \end{array} \\ \begin{pmatrix} 4.6 & 4.6 & 4.6 & 4.6 & 4.6 \\ 7.3 & 0 & 7.3 & 0 & 7.3 \\ 0 & 0 & 8.5 & 0 & 8.5 \end{pmatrix} \end{array}$$

We obtain

$$\text{Mixed rule} \rightarrow \begin{array}{c} \begin{array}{ccccc} a & b & c & d & e \end{array} \\ \begin{pmatrix} 1.7 & 1 & 8.7 & 5.7 & 6 \\ 3{,}46 & 0 & 8{,}26 & 0 & 10{,}26 \\ 0 & 0 & 8.25 & 0 & 8.75 \end{pmatrix} \end{array}$$

Computing the rules



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Up until now

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- The **axiomatic characterizations** of the rules;
- The relationship between the rules and **solution** concepts from (multi-choice) **cooperative games**;
- The **stability** of the Connection rule from a cooperative point of view (Core).

Thank You !

An Axiomatic Characterization of the Connection Rule.

Axiomatic Characterization

Axiom (Independence to Irrelevant Cost (IIC))

Pick any $(q, A^C) \in GDP$. *For each* $j \leq q_n$, *each* $i \in \hat{P}^{-1}(Q(j)) \cup Q(j)$, *and each* $\varepsilon \in \mathbb{R}$,

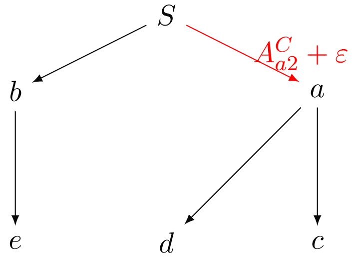
$$\forall h \in Q(j), h \notin (\hat{P}(i) \cup \{i\}),$$

$$f_{hj}(q, A^C) = f_{hj}(q, A^C + \varepsilon I^{ij}),$$

where

$$\forall k \in N, l \leq q_n, \quad I^{ij}_{kl} = \begin{cases} 1 & \text{if } k = i, l = j, \\ 0 & \text{otherwise.} \end{cases}$$

Axiomatic Characterization



$$f_{b2}(q, A^C + \varepsilon I^{a2}) = f_{b2}(q, A^C)$$
$$f_{e2}(q, A^C + \varepsilon I^{a2}) = f_{e2}(q, A^C).$$

Axiomatic Characterization

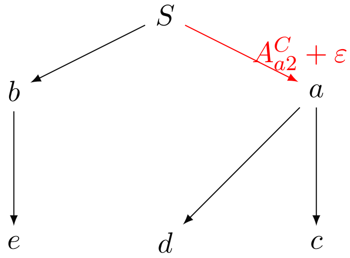
Axiom (Equal Loss for Downstream Consumers (ELD))

Pick any $(q, A^C) \in GDP$. *For each* $j \leq q_n$, *each* $i \in \hat{P}^{-1}(Q(j)) \cup Q(j)$, *and each* $\varepsilon \in \mathbb{R}$,

$$\forall h, h' \in (\hat{P}(i) \cup \{i\}) \cap Q(j),$$

$$\begin{aligned} & f_{hj}(q, A^C + \varepsilon I^{ij}) - f_{hj}(q, A^C) \\ = & f_{h'j}(q, A^C + \varepsilon I^{ij}) - f_{h'j}(q, A^C). \end{aligned}$$

Axiomatic Characterization



$$\begin{aligned} & f_{a2}(q, A^C + \varepsilon I^{a2}) - f_{a2}(q, A^C) \\ =& f_{c2}(q, A^C + \varepsilon I^{a2}) - f_{c2}(q, A^C) \\ =& f_{d2}(q, A^C + \varepsilon I^{a2}) - f_{d2}(q, A^C). \end{aligned}$$

Axiomatic Characterization

Theorem: A rule f on GDP satisfies (IIC) and (ELD)

$$\Longleftrightarrow$$

$$f = \text{Connection rule.}$$

Thank You !

Multi-Choice Games

Multi-choice games

A multi-choice game $(q, v) \in \mathcal{G}$ is given by:

- A finite player set $N = \{a, \ldots, n\}$;
- For each $i \in N$, a finite set $M_i = \{0, \ldots, q_i\}$;
- A coalition is a profile $s = (s_a, \ldots, s_n) \in \prod_{i \in N} M_i$, $q = (q_1, \ldots, q_n)$ is the grand coalition;
- A characteristic function

$$v : \prod_{i \in N} M_i \to \mathbb{R}$$

Multi-choice games

- A value is a map

$$f : \mathcal{G} \to \mathbb{R}^{\sum_{i \in N} q_i}.$$

Lowing, D. & Techer, K. (SCW 2022) introduce
φ: a generalization of the Shapley value.

Grabisch, M. & Xie, L. (MMOR 2007) introduce
Co: a generalization of the Core.

Gas distribution game

For each $(q, A^C) \in GDP$, the associated gas distribution (multi-choice) game $(q, v^{C,P})$ is defined as

$$\forall s \leq q, \quad v^{C,P}(s) = \sum_{i \in N} C(i, \overline{s}_i),$$

where

$$\forall i \in N, \quad \overline{s}_i = \max_{k \in \hat{P}(i) \cup i} s_k.$$

$v^{C,P}(s)$ is the **total cost** of a hypothetical gas distribution problem (s, A^C), where $s \leq q$.

Values and rules

For each $(q, A^C) \in GDP$,

$$\varphi(q, v^{C,P}) = \Psi(q, A^C)$$

and

$$\Psi(q, A^C) \in Co(q, v^{C,P}).$$

Thank You !

Values for multi-choice games

For each game $(q, v) \in \mathcal{G}$, the multi-choice Shapley value is defined as

$$\forall (i,j) \in M^+, \quad \varphi_{ij}(q,v) = \sum_{\substack{s \in \prod_{i \in N} M_i \\ (i,j) \in T(s)}} \frac{\Delta_v(s)}{|T(s)|}.$$

where

$$\Delta_v(s) = v(t) - \sum_{t \leq s, t \neq s} \Delta_v(t)$$

$$T(s) = \Big\{(i, s_i) \in M^+ : s_i \geq s_k,\ \forall k \in N\Big\}.$$

Values for multi-choice games

For each game $(q, v) \in \mathcal{G}$, the multi-choice Equal division value is defined as

$$\forall (i,j) \in M^+,$$
$$\xi_{ij}(q,v) = \frac{1}{|Q(j)|}\Big[v((j \wedge q_k)_{k\in N}) - v((((j-1) \wedge q_k)_{k\in N}))\Big].$$

$Q(j) = \{i \in N : q_i \geq j\}$.

Values for multi-choice games

Pick any $\alpha \in [0,1]^{q_n}$. For each $(q,v) \in \mathcal{G}$, the multi-choice Egalitarian Shapley value χ^α is defined as

$$\forall (i,j) \in M^+, \quad \chi^\alpha_{ij}(q,v) = \alpha_j \varphi_{ij}(q,v) + (1-\alpha_j)\xi_{ij}(q,v).$$

Gas distribution game

For each $(q, A^C) \in GDP$, the associated gas distribution (multi-choice) game $(q, v^{C,P})$ is defined as

$$\forall s \leq q, \quad v^{C,P}(s) = \sum_{i \in N} C(i, \overline{s}_i),$$

where

$$\forall i \in N, \quad \overline{s}_i = \max_{k \in \hat{P}(i) \cup i} s_k.$$

Each $(q, v^{C,P})$ is sub-modular, i.e.,
$v^{C,P}(s \vee t) + v^{C,P}(s \wedge t) \leq v^{C,P}(s) + v^{C,P}(t)$ for each $s, t \leq q$.

Values and rules

For each $(q, A^C) \in GDP$,

$$\begin{aligned}
\varphi(q, v^{C,P}) &= \Psi(q, A^C)\\
\xi(q, v^{C,P}) &= \Upsilon(q, A^C)\\
\chi^{\alpha}(q, v^{C,P}) &= \mu^{\alpha}(q, A^C)
\end{aligned}$$

Core and rules

The Core of a multi-choice game $(q, v) \in \mathcal{G}$ is denoted by $Co(q, v)$ and is defined as

$$x \in Co(q,v) \iff \begin{cases} \forall s \leq q, \quad \displaystyle\sum_{i \in N} \sum_{j=1}^{s_i} x_{ij} \leq v(s) \\ \forall h \leq q_n, \quad \displaystyle\sum_{i \in N} \sum_{j=1}^{h \wedge q_i} x_{ij} = v((h \wedge q_i)_{i \in N}). \end{cases}$$

Core and rules

Each sub-modular game $(q, v) \in \mathcal{G}$,

$$\varphi(q, v) \in Co(q, v).$$

NB: A game $(q, v) \in \mathcal{G}$ is sub-modular if $v(s \vee t) + v(s \wedge t) \leq v(s) + v(t)$ for each $s, t \leq q$.

Core and rules

We show that $(q, v^{C,P})$ is sub-modular, therefore

$$\varphi(q, v^{C,P}) \in Co(q, v^{C,P})$$

Thank You !

References