

# Efficient Price Discrimination for Digital Medias: The case of Publishing Brands

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## **Abstract**

After a decade during which paper press Brands have gone on-line on free or *freemium* models, new versions are emerging through smartphones and tablet devices. These versions can be offered on paying models, which might be more beneficial to publishers.

The digital scenarios are unknown and largely unpredictable; still some digital trends are the more and more evident. While willingness-to-pay for paper press is constantly lowering over generations and technological innovations, consumers have shown a willingness to pay for Medias exploited through specifically conceived digital devices.

How to capture this emerging surplus without affecting the traditional business?

The objective of this paper is to model industrial issues and pricing discrimination options emerging for publishers while developing digital versions of magazines. Moreover, we discuss policy issues emerging in the French market.

Keywords: price discrimination, digitization, publishing brands, media economics, press.



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#### Introduction

A magazine is a platform allowing for aggregation and distribution of selected copyrighted articles together with ads. It is identified through a publishing brand which signals and connotes information, adding meaning (and utility) for the consumers (Bomsel, 2011).

Publishers adjust their editing choices to capture consumers' and advertisers' surplus in a two sided market. These include, among others: optimizing the trade-off between quality and margins, developing loyalty with segmented pricing, targeting an audience including or excluding specific contents.

Digitization (Varian, Shapiro 1998) has relaxed typical constraints of publishing (space, copying and distribution), enhancing discrimination (Varian 1985, Stigler, 1987, Rochet and Stole 2002) opportunities. Moreover, it allows direct interaction with clients, which may stir up Brand popularity as well as advertising revenues.

The purpose of this paper is to analyze how efficient discrimination orients publishers' strategies through a disruptive innovation.

Discrimination issues in the digital market have been treated so far by publishers with a "by trial and error" approach, since digital scenarios are uncertain and depend on the pace of reading devices' adoption.

Before the appearance of digital reading devices, magazines' publishers have invested limitedly in the digital market. Supports such as PC's generated a high utility for news readers, due to the reduction of news' lead time, while a low utility for magazine readers, more concerned about reading comfort and quality of contents.

Opportunities and substitutability being limited, most brands have set-up a free website to generate additional advertising revenues and kept doing their traditional business as usual.

The introduction on the market of smartphones and tablets have raised the willingness to pay for written digital Medias and introduced more uncertainty on consumers' future behavior.

Investing in digital versions would be beneficial if discrimination proves to be more efficient than in the pure paper model and if cannibalization among products can be controlled.

Discriminating efficiently is further complicated since distributors of digital Medias may now compete to extract consumers surplus. Vertical integration opportunities emerge, particularly for well-known brands with large client bases.

A number of strategic questions for publishers and distributors emerge from this set-up, such as: which



type of versions and bundles<sup>1</sup> (Nalebuff, 1999, Bakos, Y. and E. Brynjolfsson. 1999) to offer? What is the optimal pricing strategy of digital versions? Is cannibalization of paper market ineluctable or can it be avoided?

These questions are tackled in the following sections of this paper, within a stylized economic framework. The economics of magazines, including paper and digital peculiarities is presented the next section. The results of the model are then presented to simulate strategic decisions. The conclusive section will tackle policy issues and the role of publishing brands in the digital magazine press.

## **Magazine Publishing**

#### 1. Publishing industry and digitization

Digitization of press magazines, radically changes the business model of the industry by relaxing both space, copying and distribution constraints. Moreover, it entails enhanced discrimination's opportunities (Varian 1985, Stigler, 1987, Rochet and Stole 2002). Two elements are particularly relevant:

- 1. Multiplication of versioning and bundling opportunities
- 2. Possibility of directly distributing contents and extending direct interaction with consumers

Both can be used as publishing tools to increase discrimination efficiency.

While the free or *freemium*<sup>2</sup> model of press websites has revealed not to be sufficiently profitable, mostly due to the redistribution of advertising revenues among Medias, the potential development of paying digital versions represent an opportunity for Media Brands to seize.

Moreover, economists have shown that when marginal cost is close to zero, bundling strategies are profit maximizing for a monopolistic firm, unless consumers have a very low degree of preference for digital goods. (Venkatesh, R. and Chatterjee, R. 2006).

The appearance of innovative devices such as smartphones and reading tablets has created a new digital market for written Medias by raising the degree of preferences and the willingness to pay for digital versions. The adoption of a new technology may also generate highly positive externalities often referred to in terms of network effects (Rohlfs 1974, Liebowitz, Margolis, 2002, Katz and Shapiro, 1986).

In order to exploit the discrimination opportunities offered by a digital market, a brand needs to produce an investment effort (production and marketing of digital versions) that varies depending on the selected

<sup>&</sup>lt;sup>2</sup> A business model that allows a consumer to receive basic services for free, but requires them to pay for any service deemed to be premium.



<sup>1</sup> Versioning and bundling are discrimination tools that will be presented in the following sections.

versioning and distributing strategy. Moreover, cannibalization<sup>3</sup> effects must be carefully evaluated.

The rate of technology adoption (potential market) and magazine-specific market share are crucial to determine the optimal digital strategy of a Publisher holding a Magazines' Portfolio.

#### 2. Paper press model: characteristics

A magazine is a (distribution) platform bundling a collection of selected copyrighted articles together with ads. The bundle is identified through a publishing brand which has specific functions: it signals and it connotes, it adds meaning for the consumer (Bomsel, 2011). It is a discrimination tool for publishers, adjusted to optimize the tradeoff between diffusion and margins.

Magazines are experience goods (Nelson 1970), meaning a utility evaluation can be made only after consumption. Moreover, publishing is a cultural industry (Baumol, William J. and William G. 1966). These concepts entail a number of peculiar characteristics:

- Uncertainty on value and quality of goods
- Infinite variety, which makes goods hard to compare.
- High concentration in traded products. A major part of sales is in best-sellers or blockbusters.
- Short life cycle. Most items are sold shortly after introduction.
- High fixed cost before introduction. There exists economy of scale in the editing sector<sup>4</sup>.
- Complex distribution structure: there is a trade-off between the number of sales points, the costs of distribution, the diffusion and the risk of unsold.

Magazines' market structure is characterized by a few major publishers, often aggregating different Brands. The rest of the market consists in a large number of agents operating on niche markets.

The cost structure of a Publisher is composed by important fixed costs, related to editing and marketing and marginal costs, related to quantity such as: cost of producing a copy, logistics and distribution. Marginal costs accounts for 30% to 50% of the cover price.

On the revenue side, publishing is a two-sided market (three sided if we included State incentives) in which both consumers and advertisers contribute to the global revenues.

#### 3. Price discrimination in the Paper Press

Versioning and bundling are editing tools which allow a publisher to increase revenues by reducing

<sup>&</sup>lt;sup>4</sup> See Caves R. (2000)



<sup>&</sup>lt;sup>3</sup> Cannibalization is the decreased demand for an existing product that occurs when its vendor releases a new and similar product.

heterogeneity in consumers' evaluation and refining price discrimination.

Versioning is the practice of selling the same content at different prices, manipulating quality and/or availability in time and space. (Varian, 1997)

Bundling is a marketing strategy, consisting in offering several products for sale as one combined product. Differential pricing techniques are particularly important when consumers have heterogeneous evaluations of a good. (Nalebuff, 1999)

The objective of these tools is to auto-sort consumers into different groups according to their willingness to pay. Consumers with high willingness to pay choose a premium offer, while consumers with lower willingness to pay auto-select a different version or bundle.

The cost structure of paper magazines limits considerably the use of these strategies. There exist special (premium) and pocket versions but the spectrum of the tool is very limited. Bundles of products are less profitable or even counter-productive if the marginal cost of goods is high (Williamson, Allen 1976). The most commonly offered bundles are:

- 1. **Bundle of issues (subscription):** consumer pays ex-ante for a definite number of issues at a generally discounted price.
- 2. **Bundles of titles:** one or more magazines are sold together with the main title as annexes.

#### 4. Digital press magazine: characteristics

A digital magazine is a digitized version of the paper magazine. The peculiar characteristic of this product is a negligible marginal cost of production and physical distribution.

This characteristic relaxes many constraints of paper model and introduces a number of interesting features for publishers (Bakos and Brynjolfsson, 1999). Digital versions may also increase consumers' utility, providing enhanced accessibility, added features and interactive contents.

Nevertheless, consumers' evaluation of digital versions has been, since their appearance, lower than evaluations of hardcopies.

The development of digital offers is costly, in term of increasing fixed costs (programming, organization and marketing costs) and emerging costs from cannibalization of the paper press.

Being substitute products (same contents) digital versions have a negative impact on paper sales. This cannibalization effect is maiden worse from the fact that it is more likely to impact very rentable markets first (high density agglomerates with broadband access), thus not lowering physical distribution costs.

Moreover, the digital model changes the interaction between customers and Media Brands introducing uncertainty for advertisers, which have started to redistribute their investments among Medias. Although



the actual gap between investments in paper ads and digital ads is likely to be reduced in the following years, the long term model is not clear yet.

Pricing a digital magazine is strictly related with strategic choices on the distribution side. While pricing a paper bundle is related to an evaluation ex-ante of the daily potential market, which is then coupled with a choice on quantity, in a digital model pricing reflects a model of auto-selection, in which each consumer access (directly or through distributors) all the contents and then selects following his personal preferences.

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#### 5. Pricing discrimination options for Digital Magazines

Digitization entails a new discrimination model that needs to be more efficient than the paper one. While publishers were exclusively in charge of discrimination in the paper model, in the digital market it can be implemented at different stages and by multiple agents.

The opportunities created by the encoding techniques are various and implementable with reasonable investments, thus magazine brands have started combining versioning and bundling techniques, following paths already exploited in other industries. Proceeding by trial and error, magazines brands have not found yet a profitable model.

#### 1. <u>Examples: Digital Versioning and Strategies</u>

- Free websites: Almost every Brand has one, offering some or the totality of the contents published in the paper magazine at no charge. The business model relies on web advertising (banners, pop-ups). In the last decade it has not produced sufficient returns to compensate cannibalization. Many Brands have recently turned from this model to freemium or premium websites.
- Premium or freemium websites. The model consists in offering reduced quality websites for free (limited contents or limited accessibility) and a paying version including totality of contents, sometimes enriched with add-ons (interactive medias, communities, interaction, archives). The Italian l'Espresso, offers basic articles for free and proposes a paying subscription giving access to extra formats and contents, such as pictures and archives of past editions. This concept have been chosen lately by top news press brands: a few months ago NYT started to offer only top news and 20 articles per month for free and requires a subscription to have unlimited access.
- Portable Document Format (PDF) is the most commonly used digital version of paper magazine. It reproduce the magazine, it can be printed in the same format or looked through with innovative devices. It has the advantage of keeping the editing of the paper model. Most publishers offer PDF versions for sale, either a single copy or a digital subscription.



- Smartphone E-reader APPS are the newest generation of digital version. They provide enhanced reading comfort and enriched contents. Most publishers developed APPS to enable the reading of their magazines on these devices. APPS development is costly and their market depends on technology adoption.
- Demo Free APPS. Applications can be manipulated easily. Some publishers offer reduced quality APPS, filled with advertising or limited in content and/or accessibility, in order to build consumer's loyalty and test his willingness to pay for the paying version.
- Really Simple Syndication. RSS is a flux of information sent from a magazine to the consumer. The subscription is usually free but it allows for accurate client targeting.
- **Timeframe.** Some publishers pose time restrictions on digital sales. This tool is mostly used to prevent cannibalization of paper sales, which are concentrated in the day of issuing of the magazine

#### 2. Examples: Bundling

- Bundle of editions (subscriptions), can be treated similarly to the paper model.
- Bundle of magazines, forming a large bouquet of contents, on the model of Pay-TV Channels Package. Digital distributors (relay.com) and associations of brands (GIE) have chosen this type of offer. The difficulty here is to find an efficient way of revenue sharing between top-brands, which are used to capture subscribers and niche-brands which my benefit from larger diffusion. Moreover, the stability of contracts is crucial.
- Bundle of versions, allowing for multi-support consumption, basically unlimited access to magazines' content in time and space. This may include access to contents on up to "four screens" (namely TV, PC, Tablet and Smartphone) plus the paper version. This type of bundle can be commercialized directly by publishing brands, lowering cannibalization and creating a digital brand. The crucial point is to be able to reorganize the company and skilled workers to a brand new economic model.

Moreover, these tools can be combined in many ways to create mixed strategies, more adapted to real economy.

#### 6. Distribution of digital magazines

In a digital environment, distribution becomes a discrimination tool as well. We consider two options for distributing digital magazines:

Outsourcing distribution: it has lower impact on business model but implies paying a distributor fee
and not having a direct interaction with clients. Unlike paper model, outsourcing distribution implies
delegating, partially or completely, discrimination (versioning and bundling) to the distributor.



 Direct distribution: it entails enhanced discrimination opportunities but involves higher investments and reengineering of brands strategy. It may result in a more efficient exploitation of brands' portfolio

Digital distributors' profits are positively correlated with the quantity and quality (in terms of market share) of Medias available on his platform.

Without a cooperative strategy, he maximizes sales exploiting available brands to create attractive offers and capture consumers' surplus. This strategy does not take in account valorization of single brands and cannibalization of paper press.

On the other hand, top-brands are must-have components of this type of offers, since they can be used as flagships to attract consumers.

Efficient revenue sharing model or vertical integration (i.e. L.P. and relay.com) should reduce this principal-agent problem.

The market of digital distribution may be slightly more competitive than in the paper model. Platform manufacturer are developing their distribution platforms trying to internalize network externalities. On the other side, publishers of must-have components may propose their own distribution platforms.

Moreover, distribution provides control on piracy. The multiplication of distributors puts more pressure on IP rights enforcement. On the other hand, the increasing market power of Platform manufacturers guarantees a higher control but may introduce lock-in cost (or even worse censorship) in proprietor networks.

#### A Model for Efficient Discrimination

Smartphones and tablets increase considerably the utility of digital versions, since they allow for enhanced accessibility and more confortable exploitation of written Medias. How can a publisher capture this utility?

Our starting point is that offering new costly versions will not conquer the public if a substitute free version is available. Moreover, paper press declining trend will continue or accelerate.

New versions have to be supported by a new business model starting from a different use of free websites, more adapted to a digital market. Less exclusive contents should be offered while more interaction must be favored.

The objective is to transform a website in a shop window, where paying versions are sold. It has to offer demo versions, signal high quality, capture new consumers. In our model, we only consider costly version, and we try to simulate the impacts of pricing and strategic decision on publisher's profits.

We try to provide guidelines to answer important strategic questions such as:



- 1. Under what product-market conditions should a publisher offer paying digital versions?
- 2. What is the optimal distribution strategy under different product-market conditions?
- 3. What assortment of digital forms bundle composition, bundle type, digital versions is optimal?
- 4. What are the implications of different pricing of paying digital version(s)?

The publisher asset is a Portfolio of brands (Magazines). Each magazine is associated with a share of the global market. This market is characterized by a few big players with relevant shares of the market (above 1%) and a large number of small players with low shares (below 0,1%). A Publisher selects a digital strategy for each brand, maximizing the overall profit. To keep it simple we do not consider here cross externalities in sales within the Portfolio.

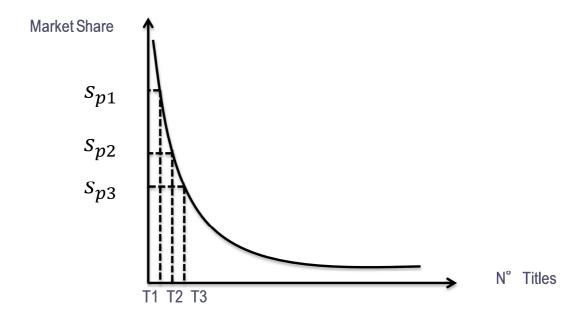


Fig 1: Representation of paper market and identification of publisher's portfolio

### 1. Definition of strategies

The model has three stages, represented by consequent binary strategic choices taken by the publisher. These choices will determine, for each Magazine, which digital strategy is adopted among the five possible solutions:

- Strategy1 ( $\pi_{ne}$ ): Not entering digital market and keep doing business as usual
- Strategy2 ( $\pi_1$ ): Outsourcing distribution of digital magazines
- Strategy3 ( $\pi_{dr}$ ): Set up a distribution platform keeping the two distributive channels separate



(paper/digital), thus becoming a direct retailer

- Strategy4 ( $\pi_{pb}$ ): Sell only a bundle composed of the paper version plus one or more digital versions of the same magazine, in a *pure bundling* strategy
- Strategy5 ( $\pi_{\rm mb}$ ): Sell both the bundle and the single versions independently, in a **mixed** bundling strategy

The strategic decision game is assembled as follows:

- 1. At t=0 the paper strategy is given. The Publisher decides whether to enter the digital market or not, in the latter case the game ends with a payoff ( $\pi_{ne}$ )
- 2. At t=1 the Publisher decides whether to distribute directly or to outsource distribution by licensing. The former choice leads to choice 3, while the latter ends up the game with output  $(\pi_1)^5$
- 3. A t=2 the Publisher chooses the marketing strategy and prices for direct distribution among three available marketing mixes:
  - Sell the digital versions independently, associated to a payoff  $(\pi_d)$
  - Create an ATAWAD6 bundle and sell only the bundle  $(\pi_{pb})$
  - Sell both digital versions and ATAWAD bundle  $(\pi_{mb})$
- 4. At t=2 if the publisher outsources, the digital distributor chooses the marketing strategy and prices. He has three options as well:
  - Sell the digital versions independently (dr)
  - Create a bundle of magazines and sell only the bundle (pb)
  - Sell both digital versions and a bundle of the two (mb)

In this phase we consider that strategies are exclusives and cannot be mixed. A brand cannot sell a multititle bundle and a digital distributor cannot sell multiform bundles.

<sup>&</sup>lt;sup>6</sup> Any Time, Any Where, Any Device



 $<sup>^{\</sup>rm 5}$  The real situation can be a mix of the two.

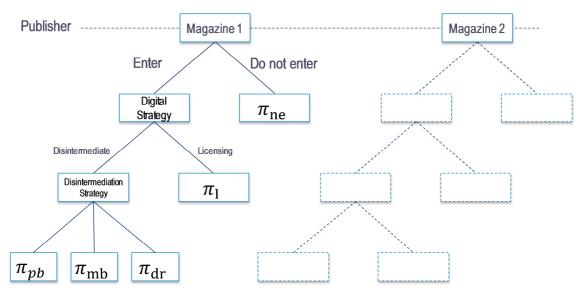


Fig 2: Sketch of the strategic decisions faced by a publisher

#### 2. Hypothesis

Different strategies have different impacts on paper sales and on cost function. Strategy1, maybe counter intuitively, is likely to have an impact on paper sales. The brand may lose competitiveness with respect to competitors and the development of the digital market may generate a reduction of paper global sales without being compensated by digital sales. Outsourcing is likely to have a strong impact on paper sales, since we are introducing in the market a substitute product and we are ceding some property rights on it. On the other hand, a multiform pure bundling strategy will have no impact since the paper version is always included in the bundle. Strategy3 will have a strong impact, intuitively at the same level of Strategy2 while Strategy5 will have a negative impact but milder than Strategy2 and 3.

Resuming, we set our first hypothesis as follows:

**Hypothesis 1:** There exists a negative covariance between digital and paper sales, this covariance is not perfectly negative and may be reduced selecting appropriate digital strategies.

Moreover, a Publisher needs to invest in order to produce and signal digital versions.

**Hypothesis 2:** Each strategy is associated to extra fixed costs for the firm: provided the paper edition already exists, these costs are a percentage of the paper fixed costs, ranging between 0 and 1. These costs are increasing with the commitment in the digital market, and they are null if a firm does not enter the market or provides licenses to sell digital editions.

Finally, we make additional hypothesis on the expected structure of the potential market:

**Hypothesis 3:** The potential market consists of surplus-maximizing potential consumers who value one or more magazines. Consumers in our model are heterogeneous in their strength of preference for each magazine. Consumers have a higher willingness to pay (wtp) for paper versions. Their wtp for a digital



version is nevertheless positive and equal to the wtp of paper version discounted by a factor  $\delta$ .

**Hypothesis 4:** Demand for paper version is linear and depends on the share of the market (exogenous for paper, we use data from OJD for the French market), this is a simple variation of demand in a monopolistic competition contest (Chamberlin, 1933).

**Hypothesis 5:** Demand for digital version depends, in addition, on development of digital market. We use the fraction of consumers equipped with a reading tablet<sup>7</sup> multiplied by the paper market value as a proxy for the digital market value. This can be refined ex post using statistics on digital magazine sales. **Hypothesis 6:** In the short term, the expected share of the digital market will be the same as the paper share for every brand.

Hypothesis 7: Elasticity to price increases in the digital market

#### 3. Inputs and outputs

The model can be used to test the impacts of digital strategies given the characteristics of a magazine brand. It may help selecting digital strategies by outlining trends, pros and cons of different options.

The inputs of the model are composed of a series of magazine specific details, most of which are collected by institutions such as OJD in France, and a set of four variables which model the expected digital scenario.

#### Magazine specific Inputs

- Market value in euro
- 2. Market share
- 3. Price elasticity
- 4. Cover price
- 5. Mean competitors price
- 6. Willingness to pay for digital version, as percentage of cover price
- 7. Production and distribution costs, as a percentage of cover price
- 8. Digital distributors' fee, as a percentage of selling price
- 9. Penetration of reading tablets in the target market, as a percentage of total readers

#### Scenario's Inputs

- 1. Expected impact of cannibalization (1=weak=-15%/-25%,2=mean,3=strong=-45%/-75%)
- 2. Expectations on the level of additional investments (1=low=+15%/+20%, 2=mean=+30%/+40%, 3=high=+45%/+60%)
- 3. Type of contract expected (in case of licensing) (from 1= favorable (coordinated pricing, min 90% of p\*) to 5=unfavorable (bundle « all you can eat », down to 50% of p\*))

<sup>7</sup> See de'Grassi 2011



4. Expected digital elasticity (1=125%paper,2=150%\*paper,3=175%\*paper)

#### **Outputs**

- 1. Theoretical optimal pricing of digital versions
- 2. Theoretical optimal digital strategy
- 3. Theoretical impacts on profits and interactions within different digital strategies

#### 4. Interesting results

#### 1. Not developing a paying digital strategy is hardly beneficial for magazines brands

The existence of a free web site is a source of cannibalization itself, the effects of which is among the causes of the negative trend in the industry. If a publisher does not invest in digital versions and competitors do, he may lose consumers with higher degree of preference for digital versions. Moreover, the development of digital press is likely to introduce positive network externalities that are going to be beneficial to competitors, further lowering the competitiveness of the publisher.

Some exception must be considered, for which this strategy may be optimal. Magazines such as:

- 1. Free press, until the gap between the willingness to pay of advertisers will be reduced
- 2. Magazines with very low diffusion and concentrated diffusion (i.e. school press, company press, religious press)
- 3. Magazines that have a target with a very low preference for digital versions (third age press)
- 4. Luxury magazines, using special types of paper and pictures which cannot be perfectly reproduced digitally

The above categories may suffer important reduction of demand or advertising revenues by producing digital versions. In such cases strategy 1 may be the optimal choice.

#### 2. <u>Direct or coordinated distribution is appealing for top-brands (subject to technology adoption)</u>

A well-known publishing brand is more likely to have the required resources to invest and will be less willing to give up a high share of the cover price and the exploitation of his brand. Moreover, he will be able to leverage a large client base as a stepping stone for the commercialization of multiform bundles. Finally, outsourcing digital distribution entails a risk of losing shares of the market by dilution in large bundles.

On the other hand, a small brand will probably have limited available resources and will thus accept to give up a higher share of his profits to have the possibility of increasing his diffusion by being in a bundle.



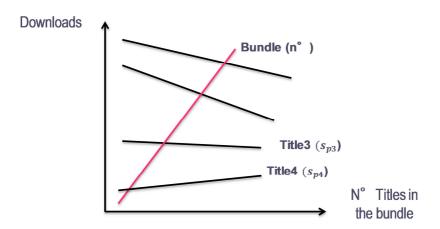


Fig 3: Externalities in an "all you can eat" type of offer

A recent survey from crmmetrix supports our intuition on consumers' behavior towards digital offers of the type known as "all you can eat" or "flat": 75% of consumers' will download magazines they would have never bought in paper version. Among this, 42% have discovered titles they didn't know and are likely to keep downloading these new magazines in the future. Finally, about 18% of consumers who discovered new titles have then bought a paper copy in the following period.

The results of the simulations, using data from the French market<sup>8</sup>, show that magazines can be divided in four categories, based on their market share in the paper market:

- 1. Top brands  $s_n \ge 1\%$  (4 % of the sample)
- 2. Strong brands . 5%  $\leq s_p \leq$  1% (13% of the sample)
- 3. Average brands  $.1\% \le s_p \le .5\%$  (41% of the sample)
- 4. Weak brands  $s_p \le .1\%$  (42% of the sample)

#### **Top brands**

When a publisher holds a top brand in his portfolio, he will have an interest to leverage this asset either by distributing digital versions directly or by using the brand as a flagship for a multiform bundle. Internalizing distribution seems to be the optimal choice for this type of brands, unless the digital markets fail to develop and becomes a niche market. In this case the investments needed to develop and signal digital offers may exceed the returns.

#### Strong brands

Strong brands have a critical market size. If they distribute directly, they may incur in important fixed cost that will be beneficial only in the mid-term with the development of the digital market. This choice may lead to a competitive advantage in the long term but it is risky. Strong brand may also try to leverage their market power to favorably negotiate licensing contracts. In this case, they may obtain positive return with

<sup>&</sup>lt;sup>8</sup> Data from OJD. The sample was composed of 349 complete entries (out of 398 total entries, excluding tv magazines).



low investments, although they may incur in progressive losses of market's share as explained above.

#### Average brands

An average magazine will hardly have the critical size to afford a digital campaign alone. In these cases, the simulations show that an optimal strategy should be outsourcing distribution. This strategy can be profitable if the licensing contract is well suited for the magazine. The development of digital market will be crucial in this case as well as the expectations on cannibalization. Counter intuitively, the development of digital market may produce lower profits for an average magazine outsourcing distribution. This is due to the coupling of the revenue sharing effect and the cannibalization of the paper press without a reduction of fixed costs.

#### Weak brands

Weak brands often rely on specific market niches. They are very sensitive to cannibalization but they should benefit from the participation in contents bundle more than the other categories. They are likely to increase their sales and benefit from flagship magazines associated with their brand. On the drawbacks, they will have a very low negotiating power vis-à-vis of distributors. The strategy of not entering the digital market at all seems to be suboptimal even for weak brands, unless they are included in the special categories described earlier in the article.

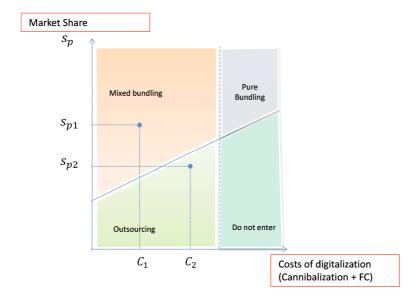


Fig 4: Visual strategic solution for given market share and investment

#### 3. Technology adoption is crucial and must be monitored and eventually subsidized

The attractiveness of digital market depends on roll-out of tablets, the latter being the optimal support for most digital versions. Recent studies have confirmed the positive trends for tablets sales worldwide, pulled by the U.S. market where tablets may already be considered as a mass technology. In France, over 1



Million consumers are equipped by now with a reading tablet. If we consider that a tablet may be used by more than one people in the same household, we have around 2 Million people who already have access to digital press out of 23 Million of total readers of press in France. This rough calculation brings us to a 8,7% of development of digital market in France in 2011.

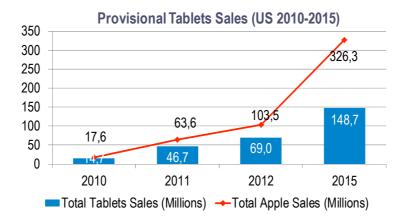


Fig 5: Latest trends on tablets adoption

According to the latest research, see graph above, this rate should almost double next year and triple again in the three years following. This means, if the growth is equally spread worldwide, that we should expect a rate of adoption (f) around 50% by 2015 for the French market. This growth will impact considerably the strategic decisions of publishers, especially for those magazines with an average to strong brand. Among these titles, those who choose to outsource distribution will profit less than the others from the growth of the digital market, thus loosing relative competitiveness. More publishers will be pushed to distribute directly.

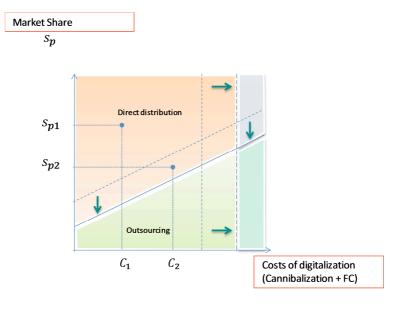


Fig 6: Effects of a growing rate of tablets diffusion



#### 4. Pricing digital versions depends on the strategy adopted

Optimal pricing of digital versions depends on the strategy (or the mix of strategies) adopted to distribute them. It should be higher than the theoretical optimal price when there is no bundle in the marketing mix, in order to push more clients towards the paper version. It should be lower when the distribution is outsourced, since the objective of the distributor is to maximize sales of his bundle without considering cannibalization problems.

#### 5. Investing in the brand to control distribution may result in a competitive advantage

The analysis of Digital press shows how profitable can be to integrate distribution in the publishers' business model. What makes digital distribution more interesting than physical one? Digital distribution is strictly correlated with the pricing of digital versions. It is not just a physical mean to reach consumers; it becomes an important marketing tool. Publishers already have a marketing division and may leverage better this opportunity by managing distribution internally.

#### 6. Bundles are useful instruments to increase profit and control cannibalization

The two types of bundles that we have analyzed have very different characteristics and effects on a publisher business model. A multiform bundle is useful to control cannibalization, regulating the switch to a digital model. It may also help publishers to drive consumers towards subscriptions and to leverage the client base to reduce the marketing costs of a digital campaign.

A contents' bundle is a way to increase digital sales and possibly the market share of a title by reducing clients' heterogeneity on magazines' evaluations. This strategy may have an opposite effect on brands with a market share slightly higher than the mean of the bundle. In this case, we can say that the strong brand is a flagship for the offer, which helps increasing sales of the bundle by leveraging the brand. This strategy may be particularly useful when a publisher holds a number of weak brands and a few flagships.

Magazine Brand	Diffusion	Market Share	Strategy	Distribution	Price digital version	Cover Price	Apps
Dossier Familial	1146319	2,57%	Mixed Bundling	Independent	,42	4,4	yes
Femme Actuelle	910207	2,04%	Digital	Licensing	1,18	1,4	yes
Notre Temps	894601	2,00%	Digital	Independent	0	3,5	no
Figaro Brand	870949	1,95%	Pure Bundling	Independent			yes
Pleine Vie	829469	1,86%	Digital	Licensing	1,65	3,3	no
Famille et Education - Magazine de l'APEL	796677	1,78%	Digital	Independent	0	0	no
Paris Match	626178	1,40%	Digital	Licensing (.com)	1,59	2,4	yes
Viva - Entre Nous La Vie	517610	1,16%	Digital	Independent	0	1,4	no
Le Nouvel Observateur	503350	1,13%	Mixed Bundling	Independent	2,51	3,5	yes
Le Particulier	461771	1,03%	Digital	Independent	0	3,9	no
Prima	458250	1,03%	Digital	Licensing	1,33	1,9	yes
Maxi	447716	1,00%	Mixed Bundling	Licensing	1,20	1,2	yes
Avantages	440167	0,99%	Mixed Bundling	Licensing	1,99	2	yes
Modes & Travaux	435236	0,97%	Mixed Bundling	Licensing	0,95	2	yes
L'Express	432418	0,97%	Digital	Independent	0	3,5	yes
Closer	431686	0,97%	Mixed Bundling	Licensing	1,2	1,5	yes
Marie Claire	431245	0,97%	Digital	Licensing	2,5	2,5	yes
Le Point	410990	0,92%	Pure Bundling	Independent	2,39	3,5	yes
Public	392011	0,88%	Digital	Licensing	1,20	1,5	yes

Fig 7: Top 20 (costly) magazines in France and their digital strategy



#### **Conclusions**

#### **Industrial Issues**

Smartphones and Tablets increase the utility of digital magazines and stir up the offer of new (digital) versions. Moreover, digitization offers enhanced discrimination opportunities. Nevertheless, the integration of these opportunities in the consolidated business model of publishers raises many issues and needs to be approached in a structured way. In the digital market, price discrimination efficiency is strictly connected with the vertical integration opportunities on the distribution side.

So far, publishers created free web-sites offering some (or all) exclusive contents for free. This model, financed by advertising, is not profitable because it cannibalizes paper sales without producing complementary returns. A free web-site can nevertheless be reconverted in a distribution platform for digital versions, offering less contents and more interaction with the brand. These platforms, besides being marketing and loyalty tools, reduce marginal costs of publishers by eliminating digital intermediaries.

The strategy of directly distributing through a proprietor platform is not suitable for every brand under today's market conditions. Most magazines have limited resources and market shares and will not be able to profitably invest to set up and signal digital versions by themself.

French Magazines can be divided in four categories, depending on the market share:

- Top brands, which seem to have a strong interest in distributing directly their digital versions, through multiform bundling offers
- 2. Strong brands, which should have either interest to distribute directly or to outsource distribution by leveraging their brand vis-à-vis of distributors, depending on the market conditions
- Average brands, which may have a stronger interest in outsourcing distribution since they have limited resources to invest
- 4. Weak brands, which are likely to have an interest in participating in large bundles in order to increase their reputation and their market share

Although above results may be modified by changes in the digital scenarios, in particular by the rate of adoption of reading tablets by the mass market, it seems already that stepping aside from the digital market is not a viable option for magazine brands.

Publishers' Portfolios are often composed of Brands from different categories. Some Publishers also have integrated physical distribution networks. Cross-externalities may then be exploited by the combination of Brands strategies. As an example, digitization provides multiple possibilities of controlling cannibalization by bundling versions and contents of magazines. By bundling different versions of a single brand, one can significantly lower the cannibalization effects. By bundling titles, one can lower heterogeneity in



consumers' evaluation to increase sales, using well-known brands as flagships to increase the diffusion of niche brands.

The development of digital market has led to the creation of new intermediaries: on the one side independent digital distributors, which can be considered as dematerialized versions of kiosks. On the other side, platform manufacturer which try to capture network externalities by centralizing distribution of contents (Apple, Amazon, Google). Publishers have thus the opportunity to exploit new intermediaries to offer digital versions without investing changing their business model.

In the latter case, they have to share profits with a distributor (similarly to paper model) and to adapt their offer to guarantee compatibility with distributor's rules. Moreover, by doing so publishers are (partially) delegating the exploitation of their brand.

The emergence of new discrimination tools and new agents capable of discriminating, coupled with the relaxation of traditional physical constraints, are the key element of the new industrial context.

This framework can be extended to newspaper brands. National newspapers are capable of connoting a large and constant information flow for their consumers. Most of them can be thus included in the category of top or strong brands.

Strategic trends can be already observed in the market. Most national newspapers are trying to circumvent the conditions imposed by global digital distributors preparing for disintermediation or negotiating tailor-made contracts with their market power. Financial Time, for example, managed to develop a direct distribution channel by exploiting a new digitized protocol (HTML5). On the other hand, a number of French brands federated in a group (e-presse) in order to gain negotiation power vis-à-vis of distributors.

Local newspaper brands may have an original business model. On the one side they are well-known in local communities (such as strong brands), on the other side they have low shares of the global market (such as medium or weak brands) and little market power bargaining with global intermediaries.

#### **Policy Issues**

Some interventions need to be made to favor investments by publishers and develop of an efficient digital market.

A key (and unsolved) issue is the monitoring and valorization of digital versions of copyrighted contents. IP enforcement costs' sharing, through the attribution of certain monitoring responsibilities to digital distributors, seems to be an option, although appropriate incentives still need to be discussed. The case of various brands recently suing research engines or distribution platform shows the need of more transparency and investments protection in many digital markets.



In the press-tablets case, since investments from Media Brands generate discrete externalities favoring the deployment of the network, enforcement cost sharing could be beneficial for both side of the market.

Finally, the normative contest in countries such as France is penalizing digital cultural industries. Publisher benefit from a reduction of VAT to 2,1% on their paper products, still they have to pay regular 19,6% VAT on digital versions. Homogenization of this and other distortions is foreseeable.



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#### **Annex**

#### Paper market

Demand:

$$(1) \quad Q_p(p_p) = S(\bar{s}_p - b_p(p_p - \bar{p}))$$

S represents the total market of magazine publishing, while  $\bar{s}_p$  is the market share of a given magazine and  $\bar{p}$  is the mean competitors' price,  $p_p$  is the cover price of the magazine and b is the elasticity.

We set:

(2) 
$$a = S(\bar{s}_p + b_p \bar{p})$$
 and  $\eta = Sb$ 

And we are back to the usual notation:

$$(3) p_p(q_p) = \frac{a - q_p}{\eta}$$

The cost function of the publishing firm is:

$$(4) c(q_p) = c_p q_p + FC_p$$

With  $FC_p$ =Fixed Costs for magazine p and:

(5) MC (Marginal Cost) = 
$$c_p$$

(6) AC (Average Cost) = 
$$\frac{FC_p}{q_p} + c_p$$

The firm maximizes his profits in the paper market:

(7) 
$$\max_{q_p} \pi_p = p_p(q_p) \times q_p - c(q_p)$$

(8) 
$$\max_{q_p} \pi_p = \left[\frac{a-q_p}{\eta} - c_p\right] \times q_p - FC_p$$

F.O.C.

(9) MR (Marginal Revenue) = MC

$$(10)\frac{\partial p_p(q_p) \times q_p}{\partial q_p} = \frac{a - 2q_p}{\eta} = MR$$

$$(11)^{\frac{a-2q_p}{n}} = c_p$$

The optimal couple is



(12) 
$$q_p^* = \frac{a - \eta c_p}{2}$$

(13) 
$$p_p^* = \frac{a + \eta c_p}{2\eta}$$

And the resulting profit is

(14) 
$$\pi_p = \frac{a^2 + (\eta c_p)^2 - 2a\eta c_p}{4\eta} - FC_p$$

#### **Digital market**

f is the fraction of consumers equipped with a reading tablet.  $S_d = fS$ .

We assume  $\bar{s}_d = \bar{s}_p$  in the short term while digital elasticity is  $b_d = b_p*(1+\epsilon)$ .

Digital demand:

(15) 
$$Q_d(p_d) = S_d(\bar{s}_d - b_d(p_d - \bar{p} + \delta))$$

Substituting  $a_d = S_d(\bar{s}_d + b_d\bar{p})$  and  $\eta = S_db_d$  and setting  $c_d = S_db_d\delta$  we have:

(16) 
$$p_d(q_d) = \frac{a_d - c_d - q_d}{\eta_d}$$

If a publisher decides to enter the digital market, he invests  $FC_d$ . We assume for simplicity that emerging costs for digital versions are a fraction of paper fixed costs

(17) 
$$c(q_d) = FC_d = \theta FC_p$$
with  $0 \le \theta \le 1$   
(18)  $MC = 0$   
(19)  $AC = \frac{FC_d}{g_d}$ 

The firm then maximizes his profits in the digital market:

$$(20) \max_{q_d} \pi_d = p_d(q_d) \times q_d - FC_d$$

(21) 
$$\max_{q_d} \pi_d = \frac{a_d - c_d - q_d}{\eta_d} \times q_d - FC_d$$

The F.O.C. for maximization is simply:

(22) 
$$MR = MC$$
  
(23)  $\frac{\partial p_d(q_d) \times q_d}{\partial q_d} = \frac{(a_d - c_d) - 2q_d}{\eta_d} = MR$   
(24)  $\frac{(a_d - c_d) - 2q_d}{\eta_d} = 0$ 

The optimal couple is



(25) 
$$q_d^* = \frac{a_d - c_d}{2}$$

(26) 
$$p_d^* = \frac{a_d - c_d}{2\eta_d}$$

And the resulting profit is

$$(27)\,\pi_d = \frac{a_d^2 + c_d^2 - 2a_d c_d}{4\eta_d} - FC_d$$

#### Strategy $3(\pi_{dr})$

Profit function:

$$(28) \pi_{dr} = \left[ \left( p_p - c_p \right) \times \left( q_p + cov_{dr}(p, d)q_d \right) - FC_p \right] + \left[ p_d q_d - FC_{dr} \right]$$

The problem is:

(30) 
$$R(q_d) = p_p(q_p + cov_{dr}q_d) + \frac{(a_d - c_d) - q_d}{\eta_d} * q_d$$
  
(31)  $C(q_d) = c_p(q_p + cov_{dr}q_d) + FC_{dr}$ 

Applying F.O.C.

$$(32) MR = \frac{\partial R(q_d)}{\partial q_d} = p_p * cov_{dr} + \frac{(a_d - c_d) - 2q_d}{\eta_d} = c_p cov_{dr} = MC$$

$$(33) q_d^* = \frac{a_d - c_d}{2} + \eta_d (p_p - c_p) * cov_{dr}$$

$$(34) p_d^* = \frac{a_d - c_d}{2\eta_d} - (p_p - c_p) * cov_{dr}$$

$$(35) \pi_{dr}^* = (p_p - c_p) \times \left(q_p + \frac{a_d - c_d}{2} * cov_{dr} + \eta_d (p_p - c_p) * cov_{dr}^2\right) + \frac{a_d^2 + c_d^2 - 2a_d c_d}{4\eta_d} - \eta_d * (p_p - c_p)^2 * cov_{dr}^2 - FC_{dr}$$

#### Strategy $4(\pi_{pb})$

Analitically we have :

$$(36) A1 = \frac{a_d - c_d}{2\eta_d} \times \frac{a}{\eta} = \frac{a(a_d - c_d)}{2\eta\eta_d}$$

$$(37) A2 = \left(\frac{a}{\eta} - \frac{a + \eta c_p}{2\eta}\right) \times \left(\frac{a_d - c_d}{\eta_d}\right) = \frac{a(a_d - c_d)}{2\eta\eta_d} - \frac{c_p(a_d - c_d)}{2\eta_d}$$

$$(38) Ab = \frac{a}{\eta} \times \left(\frac{a_d - c_d}{\eta_d}\right) - \frac{p^2_b}{2} + \frac{(p_b - \frac{a_d - c_d}{\eta_d})^2}{2\eta_d}$$

Demand for the bundle will be:

(39) 
$$Q_p(p_b) = (q_p + q_d) \times \frac{A_b}{A_1 + A_2}$$

And the resulting profit is



$$(40) \pi_b = p_b(q_b)q_b - c_p q_b - FC_b$$

To find the optimal price for the bundle we set:

(41) 
$$\alpha = \frac{A1 + A2}{q_p + q_d} = \frac{\beta - \frac{c_p(a_d - c_d)}{2\eta_d}}{\frac{a + a_d - c_d - \eta c_p}{2}}$$
 and  $\beta = \frac{a}{\eta} \times \left(\frac{a_d - c_d}{\eta_d}\right) = \frac{a(a_d - c_d)}{\eta \eta_d}$ 

Substituting in (39) and rearranging we have:

(42) 
$$p_b(q_b) = \frac{a_d - c_d}{2\eta_d} + \frac{\eta_d(\beta - \alpha q_b)}{a_d - c_d}$$

Then we set the F.O.C.

$$MR = MC$$

$$(43)\frac{a_d - c_d}{2\eta_d} + \frac{\eta_d(\beta - \alpha 2q_b)}{a_d - c_d} = c_p$$

The solution is:

$$(44) q^*_{b} = \frac{\beta}{2\alpha} + \frac{1}{\alpha} \left( \frac{a_d - c_d}{2\eta_d} \right)^2 - c_p \frac{a_d - c_d}{2\alpha\eta_d}$$

(45) 
$$p^*_b = \frac{a_d - c_d}{4\eta_d} + \frac{\eta_d \beta}{2(a_d - c_d)} + \frac{c_p}{2}$$

And the Profit will be:

$$(46)\,\pi_{pb} = \left(\frac{a_d - c_d}{4\eta_d} + \frac{\eta_d \beta}{2(a_d - c_d)} + \frac{c_p}{2}\right) \times \left(\frac{\beta}{2\alpha} + \frac{1}{\alpha} \left(\frac{a_d - c_d}{2\eta_d}\right)^2 - c_p \frac{a_d - c_d}{2\alpha\eta_d}\right) - FC_{pb}$$

#### Strategy $5(\pi_{\rm mh})$

Analitically:

$$(47) \, \mathsf{A1'} = \left(\frac{a_d - c_d}{\eta_d} - p_d\right) \times \left(p_b^* - p_d\right)$$
 
$$(48) \, \mathsf{A2'} = \left(\frac{a}{\eta} - p_p^*\right) \times \left(p_b^* - p_p^*\right)$$
 
$$(49) \, \mathsf{Ab'} = \beta + p_b^* \left(\frac{p_b^*}{2} - \frac{a}{\eta} - \frac{a_d - c_d}{\eta_d}\right) + p_p^* \left(\frac{a}{\eta} - \frac{p_p^*}{2}\right) + p_d \left(\frac{a_d - c_d}{\eta_d} - \frac{p_d}{2}\right)$$

Demand for the strategy will be:

$$(50) Q_{mb}(p_d) = q_d^* \frac{A1'(p_d)}{A1} + q_p^* \frac{A2'}{A2} + q_{pb}^* \frac{Ab'(p_d)}{A_b}$$

While the related cost function is:



(51) 
$$c(q_{mb}) = c_p \left( q_p^* \frac{A2'}{A2} + q_{pb}^* \frac{Ab'(p_d)}{A_b} \right) + FC_{mb}$$

And the resulting profit is

$$(52) \pi_{mb}(p_d) = p_b^* \left( q_b^* \frac{Ab'}{Ab} \right) + p_d \left( q_d^* \frac{A1'}{A1} \right) + p_p^* \left( q_p^* \frac{A2'}{A2} \right) - c_p \left( q_p^* \frac{A2'}{A2} + q_b^* \frac{Ab'}{Ab} \right) - FC_{mb}$$

What would be the effect of a change in  $p_d$  on publisher's profit?

$$(53)\frac{\partial \pi_{mb}}{\partial p_d} = \frac{q_d^*}{A_1} \left( 3p_d^2 - 2p_d \left( \frac{a_d - c_d}{\eta_d} + p_b^* \right) + \frac{a_d - c_d}{\eta_d} * p_b^* \right) + \frac{q_b^*}{A_b} * \left( p_b^* - c_p \right) * \left( \frac{a_d - c_d}{\eta_d} - 2p_d \right)$$

$$\text{We call } A = 3\frac{q_d^*}{A^1}, B = -\left[2\frac{q_b^*}{A_b}*\left(p_b^* - c_p\right) + 2\frac{q_d^*}{A^1}*\left(\frac{a_d - c_d}{\eta_d} + p_b^*\right)\right], C = \left(\frac{q_b^*}{A_b}*\left(p_b^* - c_p\right) + \frac{q_d^*}{A^1}*p_b^*\right) * \frac{a_d - c_d}{\eta_d}$$

The derivative is zero for:

$$(54) p_{d} = \frac{-B \pm \sqrt{B^{2} - 4AC}}{2A}$$

$$(55) p_{d,1,2} = \frac{\left[2\frac{q_{b}^{*}}{A_{b}}*(p_{b}^{*} - c_{p}) + 2\frac{q_{d}^{*}}{A_{1}}*\left(\frac{a_{d} - c_{d}}{\eta_{d}} + p_{b}^{*}\right)\right] \pm \sqrt{\left[2\frac{q_{b}^{*}}{A_{b}}*(p_{b}^{*} - c_{p}) + 2\frac{q_{d}^{*}}{A_{1}}*\left(\frac{a_{d} - c_{d}}{\eta_{d}} + p_{b}^{*}\right)\right]^{2} - 4\left[3\frac{q_{d}^{*}}{A_{1}}\right]*\left[\frac{q_{b}^{*}}{A_{b}}*(p_{b}^{*} - c_{p}) + \frac{q_{d}^{*}}{A_{1}}*p_{b}^{*}\right)*\frac{a_{d} - c_{d}}{\eta_{d}}}}{6\frac{q_{d}^{*}}{A_{1}}}}$$

